PLANE & SPHERICAL

TRIGONOMETRY

By EDWARD BROOKS, A.M. Ph.D.

PLANE AND SPHERICAL

TRIGONOMETRY.

BY

EDWARD BROOKS, A.M., PH.D.,

SUPERINTENDENT OF PUBLIC INSTRUCTION IN PHILADELPHIA;
AUTHOR OF "NORMAL SERIES OF MATHEMATICS," "NORMAL METHODS
OF TEACHING," "MENTAL SCIENCE AND CULTURE,"
"PHILOSOPHY OF ARITHMETIC," ETC.

PHILADELPHIA:
CHRISTOPHER SOWER COMPANY,
614 ARCH STREET.

WESTCOTT & THOMSON,
Sereotypers and Electrotypers, Phila.

SHERMAN & CO.,
Printers, Phila.

PREFACE.

The little work on Plane Trigonometry, written by the Author and published, in connection with his Elementary Geometry, some twenty years ago, served to introduce the subject of Trigonometry into many schools not prepared to use the larger works on that subject. This work is still well adapted to the wants of many institutions, but other schools using the writer's series of mathematics desire a more complete work on the subject.

To meet this demand, the present treatise has been prepared. It aims to furnish just so much of the subject as is taught in our best schools and colleges. Great care has been taken to give clearness and simplicity to the treatment, and to so grade the difficulties as to make the pathway of the student smooth and easy.

In treating the subject the *method of ratios*, now generally adopted by mathematicians, has been employed. The old *method of lines* is presented in the latter part of the work, but care has been taken not to mix the two methods in the development of the principles. Some of the more difficult and less practical parts of the subject are printed in smaller type, and may be omitted by

students desiring a shorter course. A large number of carefully graded problems are given to aid the student in fixing the principles and understanding their application. These exercises can be used at the option of the teacher and the requirements of the student.

In preparing the work, the best American and English works on the science have been consulted, and some valuable material has been derived from Casey and Todhunter, especially in the Exercises for the application of the principles.

PHILADELPHIA,
May 10, 1891.

EDWARD BROOKS.

CONTENTS.

HISTORY OF TRIGONOMETRY AND LOGARITHMS.

TRIGONOMETRY is believed to have originated with the Greek astronomers of Alexandria. The foundations of the science seem to have been laid by Hipparchus and Ptolemy. The first step in the science was the use of a table of chords, which served the same purpose as our table of sines. Ptolemy's celebrated work, the *Almagest*, contains a table of chords expressed in terms of the radius; and also the equivalent of several of our present formulas of trigonometry. Its treatment of spherical triangles was much more complete than that of plane triangles, which is natural, since the science was developed in the interests of astronomy.

The Indians at a very early date are known to have been familiar with the elements of the science, which they probably obtained from the Greeks. They introduced tables of half-chords, or sines, instead of chords, understood the relation between the sines and cosines of an arc and its complement, and could find the sine of half an arc from the sine and cosine of the whole arc.

The Arabs were acquainted with the *Almagest*, and probably learned from the Indians the use of the sine. Albategnius (930 A. D.) used the sine regularly, and was the first to calculate $\sin\phi$ from the formula $\sin\phi \div \cos\phi = k$. He was acquainted with the formula $\cos a = \cos b \cos c + \sin b \sin c \cos A$ for a spherical triangle ABC. Abú l'Wafá of Bagdad (b. 940) was the first to introduce the tangent as an independent function. Gheber of Seville, in the 11th century, wrote an astronomy, the first book of which contains an article on trigonometry, much in advance of that of the *Almagest*. He gave proofs of the formulas for right spherical triangles, and presented for the first time the formulæ $\cos B = \cos b \sin A$, $\cos c = \cot A \cot B$. He, however, made no advance in plane trigonometry.

Johannes Müller (1536–1476), known as Regiomontanus, wrote a treatise on the *Almagest*, in which he reinvented the tangent, and calculated a table of tangents for each degree, though he made no use of it and did not use formulæ involving the tangent. This is

said to have been the first complete European treatise on trigonometry; but its methods were in some respects behind those of the Arabs. Copernicus (1473–1543) gave the first simple demonstration of the fundamental formulæ of spherical trigonometry. George Joachim, known as Rheticus (1514–1576), wrote a work which contains tables of sines, tangents, and secants of arcs at intervals of 10″ from 0° to 90°. He found the formulæ for the sines of the half and third of an angle in terms of the sine of the whole angle.

Vieta (1540–1603) gave formulæ for the chords of multiples of a given arc in terms of the chord of the simple arc. Albert Girard (1590–1634) published a work containing theorems which gave the areas of spherical triangles, and also employed the principle of supplementary triangles. He used the notation sin, tan, sec, for the sine, tangent, and secant of an arc. Newton gave the series for an arc in terms of its sine, from which he obtained the series for the sine and cosine in powers of the arc. James Gregory in 1670 discovered the series for the arc in powers of the tangent, and for the tangent and secant in powers of the arc. Leibnitz published in 1693 the series for the sine of an arc in powers of the arc.

The greatest advance in the science was made by Euler (1707–1783), who really reduced the subject to its present condition. He introduced the present notation into general use, and made the transition from the geometrical conception of trigonometrical functions as lines to the analytical conception of functions of angles. The exponential values of sines and cosines, De Moivre's theorem, etc., are all due to Euler.

History of Logarithms.—Logarithms were invented by Lord Napier, Baron of Merchiston, in Scotland. His first work upon the subject, entitled *Mirifica Logarithorum Canonis*, was published in 1614, and gave an account of the nature of logarithms, and a table of natural sines and their logarithms for every minute of the quadrant to seven and eight figures. A second work, published after Napier's death by his son in 1619, explained the method of constructing his table. These works did not contain the logarithms of numbers, but of sines; and he called his numbers, not logarithms, but *artificials*.

Napier's system of logarithms was afterward improved by Henry Briggs, a contemporary and friend of the inventor. Assuming 10 for the basis, he constructed a system of logarithms corresponding to our system of numeration, which is much more convenient for the ordinary purposes of calculation. Briggs's first work, a small octavo tract of 16 pages, was published in 1617, and contains the first published

table of decimal or common logarithms. It gave the logarithms of numbers from unity to 1000 expressed to 14 places of decimals. A copy of the tract, now very rare, is found in the British Museum.

In 1624, Briggs published a second work, entitled *Arithmetica Logarithmica*, which contains the logarithms of numbers from 1 to 20,000, and from 90,000 to 100,000, calculated to 14 decimal places. In 1628, Adrian Vlacq, a native of Holland, published a work containing the logarithms of all numbers from 1 to 100,000. Vlacq's table is that from which all the hundreds of tables since published have been derived. It contained many errors, which have gradually been discovered and corrected; but, with one or two exceptions, no fresh calculations have ever been made.

The first publication of the common logarithms of trigonometrical functions was made in 1620 by Gunter, a colleague of Briggs in Gresham College. This work contained logarithmic sines and tangents for every minute of the quadrant to 7 decimal places. In 1633, Vlacq published a work by Briggs, entitled *Trigonometrica Britannica*, which contained logarithmic sines and tangents at intervals of a hundredth of a degree. In the same year Vlacq published his *Trigonometrica Artificialis*, giving logarithmic sines and tangents for every 10 seconds of the quadrant to 10 decimal places. These were calculated from the natural sines, etc., of the *Opus Palatinum* of Rheticus. This work fixed the method of applying logarithms to minutes and seconds, and it has never been superseded.

Napier's system of logarithms is not now in use. A modification of this system is called the Napierian, or Hyperbolic, system. It is called Hyperbolic because the logarithms represent the area of a rectangular hyperbola between its asymptotes. The base of the Napierian system is 2.718, and is denoted by the letter e. The first logarithms to the base e were published by John Speidell in 1619, in a work entitled *New Logarithms*. It contains hyperbolic log. sines, etc., for every minute of the quadrant to 5 places of decimals.

For information on centesimal logarithms, antilogarithms, logistic numbers, Gaussian logarithms, etc., see *Encyclopædia Britannica*, from which most of the above history is collated.

INTRODUCTION.

31/10/'24

THE LOGARITHMS OF NUMBERS.

1. Logarithms are a species of numbers used to abbreviate multiplication, division, involution, and evolution.

2. The **Logarithm** of a number is the exponent denoting the power to which a fixed number must be raised to produce the first number.

Thus, if $B^x = N$, then x is called the logarithm of N.

3. The **Base** of the system is the *fixed number* which is raised to the different powers to produce the numbers.

Thus, in $B^x = N$, x is the logarithm of N to the base B; so in $4^3 = 64$, 3 is the logarithm of 64 to the base 4.

4. The term *logarithm*, for convenience, is usually written *log.* The expressions above may be written $\log N = x$; and $\log 64 = 3$.

5. In the **Common System** of logarithms the base is 10, and the nature of logarithms is readily seen with this base. Thus,

$$\begin{array}{lll} 10^1 = 10; & \text{hence } \log 10 & = 1. \\ 10^2 = 100; & \text{hence } \log 100 & = 2. \\ 10^3 = 1000; & \text{hence } \log 1000 & = 3. \\ 10^{2.369} = 234; & \text{hence } \log 234 & = 2.369. \end{array}$$

6. We shall first derive the general principles of logarithms, the base being *any number*, and then explain the common numerical system.

Principles of Logarithms.

PRIN. 1. *The logarithm of* 1 *is* 0, *whatever the base.*

For, let B represent any base, then $B^0 = 1$; hence by the definition of a logarithm, 0 is the log of 1, or $\log 1 = 0$.

PRIN. 2. *The logarithm of the base of a system of logarithms is unity.*

For, let B represent any base, then $B^1 = B$; hence 1 is the log of B, or log $B = 1$.

PRIN. 3. *The logarithm of the product of two or more numbers is equal to the sum of the logarithms of those numbers.*

For, let $m = \log M$, and $n = \log N$.

Then, $B^m = M$, $B^n = N$.

Multiplying, $B^{m+n} = M \times N$.

Hence, $m + n = \log (M \times N)$.

Or, $\log (M \times N) = \log M + \log N$.

PRIN. 4. *The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.*

For, let $m = \log M$, and $n = \log N$.

Then, $B^m = M$, $B^n = N$.

Dividing, $B^{m-n} = M \div N$.

Hence, $\log (M \div N) = m - n$.

Or, $\log (M \div N) = \log M - \log N$.

PRIN. 5. *The logarithm of any power of a number is equal to the logarithm of the number multiplied by the exponent of the power.*

For, let $m = \log M$.

Then, $B^m = M$.

Raising to nth power, $B^{n \times m} = M^n$.

Whence, $\log M^n = n \times m$.

Or, $\log M^n = n \times \log M$.

PRIN. 6. *The logarithm of the root of a number is equal to the logarithm of the number divided by the index of the root.*

For, let $m = \log M$.

Then, $B^m = M$.

Taking nth root, $B^{\frac{m}{n}} = M^{\frac{1}{n}}$.

Whence, $\log M^{\frac{1}{n}} = \frac{m}{n}$.

Or, $\log M^{\frac{1}{n}} = \frac{\log M}{n}$.

7. These principles are illustrated by the following exercises, which the pupil will work.

EXERCISES.

Prove each of the following:

1. $\text{Log } (a.b.c.) = \log a + \log b + \log c.$
2. $\text{Log } \left(\dfrac{ab}{c}\right) = \log a + \log b - \log c.$
3. $\text{Log } a^n = n \log a.$
4. $\text{Log } (a^x b^y) = x \log a + y \log b.$
5. $\text{Log } \dfrac{a^x b^y}{c^z} = x \log a + y \log b - z \log c.$
6. $\text{Log } \sqrt{ab} = \frac{1}{2} \log a + \frac{1}{2} \log b.$
7. $\text{Log } (a^2 - x^2) = \log (a + x) + \log (a - x).$
8. $\text{Log } \sqrt{a^2 - x^2} = \frac{1}{2} \log (a + x) + \frac{1}{2} \log (a - x).$
9. $\text{Log } a^2 \sqrt[3]{a^{-2}} = \frac{4}{3} \log a.$
10. $\text{Log } \dfrac{\sqrt{a^2 - x^2}}{(a + x)^2} = \frac{1}{2} \{\log (a - x) - 3 \log (a + x)\}.$

Common Logarithms.

3/11/99

8. In **Common Logarithms** the base is 10. This base is most convenient for numerical calculations, because our numerical system is decimal.

9. In this system every number is conceived to be some power of 10, and by the use of fractional and negative exponents may be thus, approximately, expressed.

10. Raising 10 to different powers, we have—

$$\begin{array}{ll}
10^0 = 1; & \text{hence } 0 = \log 1. \\
10^1 = 10; & \text{hence } 1 = \log 10. \\
10^2 = 100; & \text{hence } 2 = \log 100. \\
10^3 = 1000; & \text{hence } 3 = \log 1000. \\
\text{etc.} & \text{etc.}
\end{array}$$

$$\begin{array}{lll}
\text{Also,} & 10^{-1} = .1; & \text{hence } -1 = \log .1. \\
& 10^{-2} = .01; & \text{hence } -2 = \log .01. \\
& 10^{-3} = .001; & \text{hence } -3 = \log .001.
\end{array}$$

11. Hence the logarithms of all numbers

between 1	and 10	will be	$0 +$ a fraction;
between 10	and 100	will be	$1 +$ a fraction;
between 100	and 1000	will be	$2 +$ a fraction;
between 1	and .1	will be	$-1 +$ a fraction;
between .1	and .01	will be	$-2 +$ a fraction;
between .01	and .001	will be	$-3 +$ a fraction.

12. Thus it has been found that the log of 76 is 1.8808, and the log of 458 is 2.6608. This means that

$$10^{1.8808} = 76, \text{ and } 10^{2.8608} = 458.$$

13. When the logarithm consists of an integer and a decimal, the integer is called the *characteristic*, and the decimal part the *mantissa*. Thus, in 2.660865, 2 is the *characteristic*, and .660865 is the *mantissa*.

14. The logarithm of a number less than 1 is negative; but is written in such a form that the *fractional part* is always *positive*. Thus, $\log .008 = \log (8 \times .001) = \log 8 + \log .001 = 0.903090 - 3$. Now, this may be written $\bar{3}.903090$. The minus sign is written *over* the characteristic to show that it only is negative.

Principles of Common Logarithms.

PRIN. 1. *The characteristic of the logarithm of a number is one less than the number of integral places in the number.*

For, from Art. 10, $\log 1 = 0$ and $\log 10 = 1$; hence the logarithm of numbers from 1 to 10 (which consist of *one* integral place) will have 0 for the characteristic. Since $\log 10 = 1$ and $\log 100 = 2$, the logarithm of numbers from 10 to 100 (which consist of *two* integral places) will have *one* for the characteristic, and so on; hence *the characteristic is always one less than the number of integral places.*

PRIN. 2. *The characteristic of the logarithm of a decimal is negative, and is equal to the number of the place occupied by the first significant figure of the decimal.*

For, from Art. 10, $\log .1 = -1$, $\log .01 = -2$, $\log .001 = -3$; hence the logarithms of numbers from .1 to 1 will have -1 for a characteristic; the logarithms of numbers between .01 and .1 will

have — 2 for a characteristic, and so on; hence *the characteristic of a decimal is always negative, and equal to the number of the place of the first significant figure of the decimal.*

Prin. 3. *The logarithm of the product of any number multiplied by* 10 *is equal to the logarithm of the number increased by* 1.

For, suppose $\log M = m$; then, by Prin. 3, Art. 6,

$$\log (M \times 10) = \log M + \log 10; \text{ but } \log 10 = 1;$$

Hence, $\log (M \times 10) = m + 1.$

Thus, $\log (76 \times 10) = 1.880814 + 1$; or $\log 760 = 2.880814.$

Prin. 4. *The logarithm of the quotient of any number divided by* 10 *is equal to the logarithm of the number diminished by* 1.

For, suppose $\log M = m$; then, by Prin. 4, Art. 6,

$$\log (M \div 10) = \log M - \log 10;$$

Hence, $\log (M \div 10) = m - 1.$

Thus, $\log (458 \div 10) = 2.660865 - 1$; or $\log 45.8 = 1.660865.$

Prin. 5. *In changing the decimal point of a number we change the characteristic, but do not change the mantissa of its logarithm.*

This follows from Principles 3 and 4. To illustrate:

$\log 234 = 2.369216.$ $\log .234 = \bar{1}.369216.$

$\log 23.4 = 1.369216.$ $\log .0234 = \bar{2}.369216.$

$\log 2.34 = 0.369216.$

We thus see that when we change the place of the decimal point of a number we change the characteristic, but do not change the decimal part of the logarithm.

15. Negative logarithms are sometimes written with 10 or a multiple of 10 after them, and a positive characteristic equal to the difference between its real characteristic and 10 or the given multiple of 10.

Thus, $\bar{2}.369216$ may be written $8.369216 - 10$; and $\overline{13}.369216$ may be written $7.369216 - 20$.

Table of Logarithms.

16. **A Table of Logarithms** is a table by means of which we can find the logarithms of numbers, or the numbers corresponding to given logarithms.

17. In the annexed table the entire logarithms of the numbers up to 100 are given. For numbers greater than 100 the mantissa alone is given; the characteristic being found by Prin. 1, page 12.

18. The numbers are placed in the column on the left, headed N; their logarithms are opposite, on the same line. The first two figures of the mantissa are found in the first column of logarithms.

19. The column headed D shows the average differences of the ten logarithms in the same horizontal line. This difference is found by subtracting the logarithm in column 4 from that in column 5, and is very nearly the mean or average difference.

NOTE.—The logarithms given in the Table are complete to six places. They can be readily changed to five-place logarithms by omitting the sixth figure, and when the sixth figure is 5 or more, increasing the fifth figure by 1. Similarly, we find four-place and three-place logarithms.

To Find the Logarithm of any Number.

20. *To find the logarithm of a number of* ONE *or* TWO *figures.*

Look on the first page of the table, in the column headed N, and opposite the given number will be found its logarithm. Thus,

the logarithm of 25 is 1.397940,
" " 87 is 1.939519.

21. *To find the logarithm of a number of* THREE *figures.*

Look in the table for the given number; opposite this, in column headed 0, will be found the decimal part of the logarithm, to which we prefix the characteristic 2, Prin. 1. Thus,

the logarithm of 325 is 2.511883,
" " 876 is 2.942504.

22. *To find the logarithm of a number of* FOUR *figures.*

Find the three left-hand figures in the column headed N, and opposite to these, in the column headed by the fourth figure, will be

found four figures of the logarithm, to which two figures from the column headed 0 are to be prefixed. The characteristic is 3, Prin. 1. Thus,

the logarithm of 3456 is 3.538574.
" " 7438 is 3.871456.

23. In some of the columns *small dots* are found in the place of figures: these dots mean zeros, and should be written zeros. If the four figures of the logarithm fall where zeros occur, or if, in passing back from the four figures found to the zero column, any of these *dots are passed over*, the two figures to be prefixed must be taken from the line just below. Thus,

the logarithm of 1738 is 3.240050,
" " 2638 is 3.421275.

24. *To find the logarithm of a number of* MORE THAN FOUR *figures.*

Place a decimal point after the fourth figure from the left hand, thus changing the number into an integer and a decimal. Find the mantissa of the entire part by the method just given. Then from the column headed D take the corresponding *tabular difference*, multiply it by the decimal part, and add the product to the mantissa already found: the result will be the mantissa of the given number. The characteristic is determined by Prin. 1.

If the decimal part of the product exceeds .5, we add 1 to the entire part: if less than .5, it is omitted.

EXERCISES.

1. Find the logarithm of 234567.

SOLUTION.—The characteristic is 5, Prin. 1. Placing a decimal point after the fourth figure from the left, we have 2345.67. The decimal part of the logarithm of 2345 is .370143: the number in column D is 185: and $185 \times .67 = 123.95$, and since .95 exceeds .5, we have 124, which, added to .370143, gives .370267: hence, log $234567 = 5.370267$.

Find the logarithm

2. Of 4567.	*Ans.* 3.659631.	8. Of 704.307.	*Ans.* 2.847762.
3. Of 3586.	*Ans.* 3.554610.	9. Of .000476.	*Ans.* $\bar{4}$.677607.
4. Of 11806.	*Ans.* 4.072102.	10. Of $\frac{365}{400}$.	*Ans.* $\bar{1}$.960233.
5. Of .4729.	*Ans.* $\bar{1}$.674769.		
6. Of 29.337.	*Ans.* 1.467416.	11. Of $\frac{375}{463}$.	*Ans.* $\bar{1}$.908450.
7. Of 734582.	*Ans.* 5.866040.		

NOTE.—To find the logarithm of a common fraction, subtract the logarithm of the denominator from the logarithm of the numerator.

25. *To find the number corresponding to a given logarithm.*

1. Find the *two left-hand* figures of the *mantissa* in the column headed 0, and the other four, if possible, in the same or some other column on the same line; then, in column N, opposite to these latter figures, will be found the *three left-hand* figures, and at the top of the page the other figure of the required number.

2. *When the exact mantissa is not given in the table,* take out the four figures corresponding to the *next less mantissa* in the table; subtract this mantissa from the given one; divide the remainder, with ciphers annexed, by the number in column D, and annex the quotient to the four figures already found.

3. Make the number thus obtained correspond with the characteristic of the given logarithm, by pointing off decimals or annexing ciphers.

EXERCISES.

1. Find the number whose logarithm is 5.370267.

SOLUTION.—The mantissa of the given logarithm is 370267
The mantissa of the next less logarithm of the table is . . .370143
and its corresponding number is 2345.
Their difference is 124
The tabular difference is 185
The quotient is . . . . 185)124.00(.67
Hence the required number is 234567

NOTE.—If the characteristic had been 2, the number would have been 234.567; if it had been 7, the number would have been 23456700; if it had been $\bar{2}$, the number would have been .0234567, etc.

Find the number whose logarithm

2. Is 3.659631.	*Ans.* 4567.	7. Is 4.790285.	*Ans.* 61700.
3. Is 3.563125.	*Ans.* 3657.	8. Is $\bar{2}$.674769.	*Ans.* .04729.
4. Is 2.554610.	*Ans.* 358.6.	9. Is $\bar{3}$.065463.	*Ans.* .0011627.
5. Is 1.072102.	*Ans.* 11.806.	10. Is $\bar{3}$.514548.	*Ans.* .00327.
6. Is 4.883150.	*Ans.* 76410.	11. Is $\bar{4}$.846741.	*Ans.* .00070265.

Multiplication by Logarithms.

26. From Prin. 3, for the multiplication of numbers by means of logarithms, we have the following

RULE.—*Find the logarithms of the factors, take their sum, and find the number corresponding to the result; this number will be the required product.*

NOTE.—The term *sum* is used in its algebraic sense. Hence, when any of the characteristics are negative, we take the difference between the sums of the positive and negative characteristics, and prefix to it the sign of the greater. If anything is to be "carried" from the addition of the mantissa, it must be added to a positive characteristic or subtracted from a negative one.

When any of the characteristics are negative, we can write them as suggested in Art. 15, and proceed accordingly.

EXERCISES.

1. Multiply 35.16 by .815.

SOLUTION.—

$$\begin{aligned} \log 35.16 &= 1.546049 \\ \log\ .815 &= \bar{1}.911158 \\ & \quad\ \ 1.457207 \\ & \quad\ \ \ \ 457125 \end{aligned}$$

Product, 28.6554   152)82.00(.54

Find the product

2. Of .7856, 31.42. *Ans.* 24.6835.
3. Of 0.3854 by 0.0576. *Ans.* .022199.
4. Of 31.42, 56.13, and 516.78. *Ans.* 911393.7.
5. Of 31.462, .05673, and .006785. *Ans.* .01211168.
6. Of .06517, 2.16725, .000317, and 42.1234. *Ans.* .001886.
7. Of 2.3456, .00314, 123.789, .00078, and 67.105. *Ans.* .04772076.

Division by Logarithms.

27. From Prin. 4, to divide by means of logarithms, we have the following

Rule.—*Find the logarithms of the dividend and divisor, subtract the latter from the former, and find the number corresponding to the result; this number will be the required quotient.*

Note.—The term *subtract* is here used in its algebraic sense: hence, when any of the characteristics are negative we must subtract according to the principles of algebra.

Negative characteristics may be written as in Art. 15, for subtraction.

EXERCISES.

1. Divide 783.5 by .625.

First Solution.		Second Solution.
$\log 783.5 = 2.894039$		$\log 783.5 = 12.894039 - 10$
$\log \ .625 = \bar{1}.795880$		$\log \ .625 = 9.795880 - 10$
3.098159		Dif. $= 3.098159$
Quo. 1253.6	$.097951$	Quo. 1253.6.
	346)208(6	

2. Divide 272.636 by 6.37. *Ans.* 42.8.
3. Divide 50.38218 by 67.8. *Ans.* .7431.
4. Divide 155 by .0625. *Ans.* 2480.
5. Divide 1.1134 by 0.225. *Ans.* 5.04.
6. Divide 0.10071 by 0.00373. *Ans.* 27.
7. Divide 435×684 by 583×760. *Ans.* 671524.

The Cologarithm of a Number.

28. The **Cologarithm** of a number is the result arising from subtracting the logarithm of the number from 10. Thus, colog $N = 10 - \log N$, and colog $40 = 10 - \log 40$, or $10 - 1.60206 = 8.39794$.

29. The cologarithm of a number may be written directly from the table by subtracting each term of the logarithm from 9, except the right-hand term, which must be taken from 10.

30. The cologarithm is used to simplify the operation of division

when it is combined with multiplication. Thus, suppose we wish to divide M by N.

Now, $\log(M \div N) = \log M - \log N$.

But, $\log N = 10 - \operatorname{colog} N$. Art. 28.

Substituting, $\log M - \log N = \log M + \operatorname{colog} N - 10$.

Hence, instead of *subtracting* log N, we may *add* colog N, and then deduct 10 from the sum.

31. Hence, to divide by means of the cologarithm of a number we have the following

RULE.—*Add the cologarithm of the divisor to the logarithm of the dividend, subtract* 10, *and find the number corresponding to the result.*

NOTE.—The cologarithm is sometimes defined as the *logarithm of the reciprocal of the number*, and the rule for its use deduced accordingly. The cologarithm as defined above is usually known as the *Arithmetical Complement.*

EXERCISES.

1. Divide 256.3 by 45.32.

SOLUTION.—	log 856.3	2.932626
	colog 45.32	8.343710
Quotient,	18.8945	1.276336

2. Divide 0.3156 by 78.35.

	log 0.3156	$\bar{1}.499137$
	colog 78.35	8.105961
Quotient,	.004028	$\bar{3}.605098$

3. Divide 3.7521 by 18.346. *Ans.* .204519.
4. Divide 483.72 by .30751. *Ans.* 1573.02.
5. Find value of $32.16 \times 7.856 \div 45.327$. *Ans.* 5.574.
6. Of 31.57×123.4 divided by $316.2 \times .0316$. *Ans.* 389.8884.
7. Of x, given $x : 73.15 = 40.16 : 3167$. *Ans.* 1.11237.
8. Of x, given $72.34 . 2.519 = 357.48 : x$. *Ans.* 12.448.

Involution by Logarithms.

32. From Prin. 5, to raise a number to any power, we have the following

RULE.—*Find the logarithm of the number, multiply it by the exponent of the power, and find the number corresponding to the result.*

EXERCISES.

1. Find the 4th power of 45.

SOLUTION.—

$$\begin{array}{lrr} & \log 45 = & 1.653213 \\ & & 4 \\ \hline \text{Power,} & 4100625 & 6.612852 \end{array}$$

2. Find the cube of 0.65. *Ans.* 0.2746.
3. Find the 6th power of 1.037. *Ans.* 1.243.
4. Find the 7th power of .4797. *Ans.* 0.005846.
5. Find the 30th power of 1.07. *Ans.* 7.6123.

Evolution by Logarithms.

33. From Prin. 6, to extract from any root of a number, we have the following

RULE.—*Find the logarithm of the number, divide it by the index of the root, and find the number corresponding to the result.*

NOTE.—If the characteristic is NEGATIVE, and NOT DIVISIBLE by the index of the root, add to it the smallest negative number that will make it divisible, prefixing the same number with a plus sign to the mantissa.

EXERCISES.

1. Find the square root of 576.

SOLUTION.—

$$\log 576 = 2.760422$$
$$2.760422 \div 2 = 1.380211$$

Hence, the root is 24.

2. Find the fourth root of .325.

SOLUTION.—

$$\begin{array}{rr} \log .325 = & \bar{1}.511883 \\ -3 & +3 \\ \hline 4)\,-4 + & 3.511883 \\ \hline & \bar{1}.877971 \end{array}$$

Hence, the root is, .75504.

3. Find the cube root of 7. *Ans.* 1.9129.
4. Find the fifth root of 5. *Ans.* 1.3797.
5. Find the fifth root of .0625. *Ans.* .574348.
6. Find the seventh root of 7. *Ans.* 1.32047.
7. Find the tenth root of 8764.5. *Ans.* 2.479.

Calculation of Logarithms.

The pupil will naturally desire to know how these logarithms are calculated. While this is not the place to enter into a detailed explanation of the method of calculating logarithms, a general idea of the subject can be presented.

In computing logarithms it is necessary to calculate only the logarithms of prime numbers, since the logarithms of composite numbers may be obtained by adding the logarithms of their prime factors.

The logarithms of the prime numbers were first computed by comparing the geometrical and arithmetical series, 1, 10, 100, etc., and 0, 1, 2, etc., and finding geometrical and arithmetical means; the arithmetical mean being the logarithm of the corresponding geometrical mean. This method was exceedingly laborious, involving so many multiplications and extractions of roots.

The method now generally used is that of series, by which the computations are much more easily made. The following formula is derived by algebraic reasoning:

$$\log (1+x) = A\left(\frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \text{etc.}\right).$$

In this series the quantity A is called the *modulus*, which in the Napierian system is *unity*. The series, when A is *one*, put in a more convenient form, becomes

$$\log (z+1) - \log z = 2\left(\frac{1}{2z+1} + \frac{1}{3(2z+1)^3} + \frac{1}{5(2z+1)^5} + \text{etc.}\right).$$

From which, knowing the logarithm of any number, we readily find the logarithm of the next larger number. The student will be interested in finding logarithms by this formula. Begin with 2, in which $z = 1$.

The logarithm found will be the Napierian logarithm, and this multiplied by 0.434294 will give the common logarithm.

Logarithms were invented by Lord Napier of Scotland, and are regarded as among the most useful inventions ever made. His system was subsequently improved by Henry Briggs, a cotemporary of Napier's, who, assuming 10 for a basis, constructed a system much more convenient for the ordinary purposes of computation. Napier's system was also modified by John Speidell, whose logarithms are now known as the Napierian or Hyperbolic logarithms. Briggs' logarithms are known as the Briggean or Common logarithms.

It is generally believed that the so-called "Napierian logarithms" are identical with those first computed by Napier; but this is not the case. For a more detailed statement of the origin of logarithms, see the History of Logarithms given in the Introduction of this work, page 6.

PLANE TRIGONOMETRY.

SECTION I.

THE MEASUREMENT OF ANGLES.

1. **Trigonometry** is the science which investigates the relation of the sides and angles of triangles.

2. **Plane Trigonometry** treats of plane angles and triangles; **Spherical Trigonometry** treats of spherical angles and triangles.

3. In every triangle there are six parts, three sides and three angles. These parts are so related that when certain ones are given, the others may be found.

4. In Geometry the triangle can be *constructed* when a sufficient number of parts are given. In Trigonometry the unknown parts are *computed* from the known parts.

5. In order to subject a triangle to computation, we must be able to express its sides and angles by numbers. For this purpose proper units must be adopted.

6. The units of measure for the sides are straight lines of a fixed length, as the *inch*, *foot*, *yard*, etc. The units of measure for angles are *degrees*, *minutes*, and *seconds*.

NOTE.—Trigonometry is really a *numerical* way of treating triangles in distinction from the *geometrical* way of treating them. The science extends also to the investigation of angles in general, and is then called *Angular Analysis*.

Measures of Angles.

7. An angle is measured, as shown in geometry, by the arc intercepted between its sides, the centre of the circle being at the vertex of the angle.

8. The units of the arc are equal parts of the circumference called *degrees*, *minutes*, and *seconds*. A *degree* (marked °) is $\frac{1}{360}$ of the circumference; a *minute* (marked ′) is $\frac{1}{60}$ of a degree; and a *second* (marked ″) is $\frac{1}{60}$ of a minute.

9. A **Quadrant** is one-fourth of the circumference of a circle. Each quadrant contains 90°, and is the measure of a right angle.

10. Reckoning from A, the arc AB is called the *first quadrant;* the arc BC the *second quadrant;* the arc CD the *third quadrant;* the arc DA the *fourth quadrant.* The term quadrant is also applied in the same manner to the four equal parts of the circle.

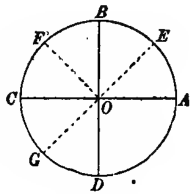

Fig. 1.

11. Any arc AE, less than 90°, is said to be in the first quadrant; any arc AF, between 90° and 180°, is said to be in the second quadrant; any arc AG, between 180° and 270°, in the third quadrant, etc.

12. The **Complement** of an angle or an arc is the remainder obtained by subtracting the angle or arc from 90°. Thus, the complement of arc AE is arc BE.

13. The **Supplement** of an angle or an arc is the remainder obtained by subtracting the angle or arc from 180°. Thus, the supplement of arc AF is arc CF.

14. According to these definitions, the complement of an arc greater than $90°$ is negative, and the supplement of an arc greater than $180°$ is negative. Thus, the complement of $120°$ is $90° - 120° = -30°$; and the supplement of $200°$ is $180° - 200° = -20°$.

Numerical Lengths of Arcs.

15. The units of the circle—that is, degrees, minutes, and seconds—express equal parts of the circumference. An arc may also be expressed in numerical units corresponding to a straight line.

I. *To find a numerical expression for an arc of a given number of degrees, minutes, etc.*

The circumference of a circle is $2\pi R$ (B. V., Th. 8). Supposing $R = 1$, we find the semi-circumference equal to $\pi = 3.14159265$. Hence,

Arc $180° = 3.14159265$.	Arc $1' = 0.000290888$.
Arc $1° = 0.01745329$.	Arc $1'' = 0.000004848$.

II. *To find the number of degrees, minutes, etc., in an arc equal to the radius.*

Since, $\quad 2\pi R = 360°, \quad \pi R = 180°.$

$$\text{Hence,} \quad R = \frac{180°}{\pi} = \frac{180°}{3.14159265} = 57°.2957795.$$

$$= 3437'.74677 = 206264''.806.$$

16. The angle at the centre measured by an arc equal to the radius, it is thus seen, is an *invariable angle*, whatever the length of the radius; hence it is often taken as the unit of angular measure.

17. Since when the radius is unity, $2\pi = 360°$, π is often used to express two right angles. Then $\frac{\pi}{2}$ equals a right

angle; 2π equals four right angles; $\frac{\pi}{4}$ = an angle of $45°$, etc.

18. This method of measuring an angle is called the *circular measure* of an angle. The method by *degrees*, etc. is called the *sexagesimal method.*

NOTE.—A third method, called the *centesimal* method, was proposed by the French at the introduction of the metric system. In this system the right angle was divided into 100 parts, called *grades*, each grade into 100 parts called *minutes*, etc.

EXERCISES I.

1. How many degrees in an angle denoted by 2π? By π? By $\frac{1}{2}\pi$? By 3π? By $\frac{3}{4}\pi$? By $\frac{1}{8}\pi$? By $\frac{3}{16}\pi$? $n\pi$?

2. Express in terms of π an angle of $180°$; of $90°$; of $60°$; of $45°$; of $30°$; of $70°$; of $80°$; of $63°$; of $67° 30'$; of $52° 30'$.

3. How many degrees in an arc whose length is equal to the diameter of the circle? *Ans.* $114°. 59 +$.

4. How many degrees in an arc whose length is 0.6684031? Whose length is 2.0052093? *Ans.* $38° 17' 48''$; $114° 53' 24''$.

5. Express $\frac{5}{16}$ of a right angle in degrees and minutes; also in circular measure. *Ans.* $28° 7'.5$; $\frac{5}{32}\pi$.

6. What is the length of an arc of $60°$ when the radius is 8? When the radius is 12? Radius 20?

7. When the radius is 8, required the length of an arc of $45°$; of $75°$; of $22° 30'$; of $52° 30'$; of $33° 45'$.

8. Find the diameter of a globe when an arc of a great circle of $25°$ measures 4 feet. *Ans.* 18.3346.

9. Find the number of degrees in a circular arc 30 inches in length, the radius being 25 inches. *Ans.* $68° 45' 17'' +$.

SECTION II.

TRIGONOMETRICAL FUNCTIONS.

19. In **Trigonometry**, instead of comparing the angles of triangles or the arcs which measure them, we compare certain lines or ratios of lines called the *functions* of the angles.

20. A *Function* of a quantity is something depending on the quantity for its value. These functions in Trigonometry are the *sine*, *cosine*, *tangent*, *cotangent*, *secant*, and *cosecant.*

21. Since every oblique triangle can be resolved into two right triangles by drawing a perpendicular from one of its angles to the opposite side, the solution of all triangles can be made to depend on that of right triangles.

22. The functions, *sine*, *cosine*, etc., are used to express the relation of the sides of the right triangle. These terms will now be defined and illustrated.

23. In the right triangle ABC, let AC be denoted by b, BC by a, and AB by c; then we have the following definitions:

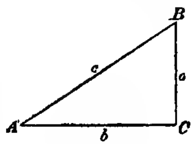

Fig. 2.

1. *The* **Sine** *of an angle is the ratio of the* ***opposite*** *side to the* ***hypotenuse.***

Thus, $\sin A = \dfrac{BC}{AB} = \dfrac{a}{c}$; $\sin B = \dfrac{AC}{AB} = \dfrac{b}{c}$.

2. *The* **Tangent** *of an angle is the ratio of the* ***opposite*** *side to the* ***adjacent*** *side.*

Thus, $\tan A = \dfrac{BC}{AC} = \dfrac{a}{b}$; $\tan B = \dfrac{AC}{BC} = \dfrac{b}{a}$.

3. *The* **Secant** *of an angle is the ratio of the* ***hypotenuse*** *to the* ***adjacent*** *side.*

Thus, $\sec A = \frac{AB}{AC} = \frac{c}{b}$; $\sec B = \frac{AB}{BC} = \frac{c}{a}$.

4. *The* **Cosine** *of an angle is the sine of the complement of the angle.*

Thus, $\cos A = \sin B = \frac{b}{c}$; hence $\cos A = \frac{b}{c}$.

Also, $\cos B = \sin A = \frac{a}{c}$; hence $\cos B = \frac{a}{c}$.

5. *The* **Cotangent** *of an angle is the tangent of the complement of the angle.*

Thus, $\cot A = \tan B = \frac{b}{a}$; $\cot B = \tan A = \frac{a}{b}$.

6. *The* **Cosecant** *of an angle is the secant of the complement of the angle.*

Thus, $\csc A = \sec B = \frac{c}{a}$; $\csc B = \sec A = \frac{c}{b}$.

24. If A denotes any angle or arc, then we have from the above explanations,

$$\sin A = \cos (90° - A); \quad \cos A = \sin (90° - A).$$
$$\tan A = \cot (90° - A); \quad \cot A = \tan (90° - A).$$
$$\sec A = \csc (90° - A); \quad \csc A = \sec (90° - A).$$

NOTE.—The above definitions of cosine, cotangent, and cosecant show their true relation to the sine, tangent, and secant. We may, however, define them independently, as follows:

1. *The cosine of an angle is the ratio of the* ***adjacent*** *side to the* ***hypotenuse.***

2. *The cotangent of an angle is the ratio of the* ***adjacent*** *side to the* ***opposite*** *side.*

3. *The cosecant of an angle is the ratio of the* ***hypotenuse*** *to the* ***opposite*** *side.*

25. The sine, cosine, tangent, cotangent, etc. are called *Trigonometrical Functions* or *Ratios.* A large part of Trigonometry consists in the investigation of the properties and relations of these functions of an angle.

NOTE.—If the cosine of an angle is subtracted from unity, the remainder is called the *versed-sine* of an angle; if the sine of an angle is subtracted from unity, the remainder is called the *coversed-sine* of the angle.

Thus, $\text{vers } A = 1 - \cos A$; $\quad \text{covers } A = 1 - \sin A$.

EXERCISES II.

1. Find the values of the trigonometrical functions of A, when $a = 3$, $b = 4$, and $c = 5$.

SOLUTION.—By Art. 23,

$$\sin A = \frac{a}{c} = \frac{3}{5};\ \cos A = \frac{b}{c} = \frac{4}{5};\ \tan A = \frac{a}{b} = \frac{3}{4};\ \text{etc.}$$

2. Find the values of the trigonometrical functions of A when $a = 5$, $b = 12$, and $c = 13$. When $a = 8$, $b = 15$, and $c = 17$.

3. Write all the functions of B in the triangle of Fig. 2.

4. Find the functions of A and of B when $a = 10$ and $b = 24$. When $a = 18$ and $c = 82$. When $b = 75$ and $c = 85$.

5. Find the functions of A and of B when $a = m$ and $c = m\sqrt{2}$. When $b = \sqrt{2mn}$ and $c = m + n$.

6. Find a, if $\sin A = \frac{3}{4}$ and $c = 5$. Find b, if $\tan A = \frac{5}{12}$ and $c = 13$. Find c, if $\sec A = 2$ and $b = 5$.

7. Compute the functions of A when $a = \frac{3}{4}b$. When $b = \frac{4}{5}c$. When $a + b = \frac{5}{4}c$. When $a - b = \frac{1}{4}c$.

8. Construct a right triangle when $\sin A = \frac{3}{5}$ and $a = 9$. When $\tan A = \frac{2}{3}$ and $b = 9$. When $\csc A = 3.5$ and $c = 4\frac{1}{2}$.

9. Compute the legs of a right triangle when $\sin A = 0.4$, $\cos A = 0.6$, and $c = 4.5$. Construct the triangle.

10. Given $A + B = 90°$; to prove the following:

$\sin A = \cos B.$	$\tan A = \cot B.$	$\sec A = \csc B.$
$\cos A = \sin B.$	$\cot A = \tan B.$	$\csc A = \sec B.$

Fundamental Formulas.

26. We now proceed to derive some fundamental formulas expressing the relations of trigonometrical functions.

I. *Formulas expressing the relation of sine and cosine.*

1\. In the right triangle ABC, by geometry,

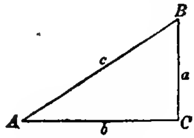

Fig. 3.

We have, $a^2 + b^2 = c^2.$

Whence, $\dfrac{a^2}{c^2} + \dfrac{b^2}{c^2} = 1.$

Substituting the values of sin A and cos A (Art. 23),

We have, $$\sin^2 A + \cos^2 A = 1. \qquad [1]$$

That is: *The sum of the squares of the sine and cosine of an angle is equal to unity.*

Note.—We write $\sin^2 A$ for $(\sin A)^2$ and $\cos^2 A$ for $(\cos A)^2$ as a matter of convenience, and similarly with the powers of the other trigonometrical functions.

2\. From the above formula we have

$$\sin^2 A = 1 - \cos^2 A = (1 + \cos A)(1 - \cos A).$$
$$\cos^2 A = 1 - \sin^2 A = (1 + \sin A)(1 - \sin A).$$

II. *Formulas expressing the relation of tangent and cotangent.*

1\. From Art. 23, (1) and (4), we have

$$\frac{\sin A}{\cos A} = \frac{a}{c} \div \frac{b}{c} = \frac{a}{b}; \text{ but } \tan A = \frac{a}{b}. \qquad \text{Art. 23, (2).}$$

Whence, $$\tan A = \frac{\sin A}{\cos A}. \qquad [2]$$

That is: *The tangent of an angle is equal to the sine divided by the cosine.*

2. From Art. 23, (5), we have

$$\cot A = \frac{b}{a}; \quad \text{but} \quad \frac{\cos A}{\sin A} = \frac{b}{c} \div \frac{a}{c} = \frac{b}{a}.$$

Whence, $$\cot \mathrm{A} = \frac{\cos \mathrm{A}}{\sin \mathrm{A}}. \qquad [3]$$

That is: *The cotangent of an angle is equal to the cosine divided by the sine.*

3. Taking the product of [2] and [3], we have

$$\tan A \times \cot A = 1.$$

Whence, $$\tan \mathrm{A} = \frac{1}{\cot \mathrm{A}}; \quad \text{or} \quad \cot \mathrm{A} = \frac{1}{\tan \mathrm{A}}. \qquad [4]$$

That is: *The tangent and the cotangent of an angle are reciprocals of each other.*

III. *Formulas expressing the relation of secant and cosecant to the other functions.*

1. From Art. 23, $\sin A = \frac{a}{c}$ and $\csc A = \frac{c}{a}$.

Whence, $$\sin \mathrm{A} = \frac{1}{\csc \mathrm{A}}; \quad \text{or} \quad \csc \mathrm{A} = \frac{1}{\sin \mathrm{A}}. \qquad [5]$$

That is: *The sine and cosecant of an angle are reciprocals of each other.*

2. From Art. 23, $\cos A = \frac{b}{c}$ and $\sec A = \frac{c}{b}$.

Hence, $$\cos \mathrm{A} = \frac{1}{\sec \mathrm{A}} \quad \text{and} \quad \sec \mathrm{A} = \frac{1}{\cos \mathrm{A}}. \qquad [6]$$

That is: *The cosine and secant of an angle are reciprocals of each other.*

3. In the right triangle ABC

We have $$c^2 = b^2 + a^2.$$

Dividing by b^2, $$\frac{c^2}{b^2} = 1 + \frac{a^2}{b^2}.$$

Whence, $$\sec^2 \mathrm{A} = 1 + \tan^2 \mathrm{A}. \qquad [7]$$

In a similar manner we find

$$\csc^2 A = 1 + \cot^2 A.$$

27. These expressions derived under Art. 26 may be regarded as the Fundamental Formulas of Trigonometry, and should be committed to memory. We shall collect them, forming the following table:

TABLE I.

1. $\text{Sin}^2 A + \cos^2 A = 1$	8. $\text{Cot}\ A = \dfrac{1}{\tan A}$
2. $\text{Sin}^2 A = 1 - \cos^2 A$	9. $\text{Sec}\ A = \dfrac{1}{\cos A}$
3. $\text{Cos}^2 A = 1 - \sin^2 A$	10. $\text{Csc}\ A = \dfrac{1}{\sin A}$
4. $\text{Tan}\ A = \dfrac{\sin A}{\cos A}$	11. $\text{Sec}^2 A = 1 + \tan^2 A$
5. $\text{Cot}\ A = \dfrac{\cos A}{\sin A}$	12. $\text{Csc}^2 A = 1 + \cot^2 A$
6. $\text{Tan}\ A \cot A = 1$	13. $\text{Ver sin}\ A = 1 - \cos A$
7. $\text{Tan}\ A = \dfrac{1}{\cot A}$	14. $\text{Co-ver sin}\ A = 1 - \sin A$

NOTE.—1. The student should be able to state these formulas and also express them in theorems.

2. The student should also fix the following truths in his understanding:

(*a*) *Either side of a right triangle equals the hypotenuse into the sine of the opposite angle.*

(*b*) *Either side of a right triangle equals the hypotenuse into the cosine of the adjacent angle.*

EXERCISES III.

Find the values of the other functions, when

1. $\sin A = \frac{3}{5}$.
2. $\sin A = \frac{5}{13}$.
3. $\cos A = \frac{4}{5}$.
4. $\cos A = \sqrt{\frac{2}{3}}$.
5. $\tan A = \frac{5}{12}$.
6. $\tan A = 2$.
7. $\cot A = \frac{3}{4}$.
8. $\cot A = \frac{2}{3}$.
9. $\sec A = \frac{3}{2}$.
10. $\csc A = \sqrt{5}$.
11. $\sin A = \dfrac{2n}{1 + n^2}$.
12. $\cos A = \sqrt{1 - n^2}$.

Find the value of the other functions:

13. Given $\sin 30° = \frac{1}{2}$.
14. Given $\sin 45° = \frac{1}{2}\sqrt{2}$.
15. Given $\sec 60° = 2$.
16. Given $\tan 45° = 1$.
17. Given $\sin 90° = 1$.
18. Given $\sec 45° = \sqrt{2}$.

Find the other functions from the following equations:

19. $\sin A = 2 \cos A$.
20. $\sin A = \frac{3}{2} \cos A$.
21. $\tan A = \frac{1}{2} \sec A$.
22. $\tan A = 4 \cot A$.
23. $\tan A = m \sin A$.
24. $\sec A = n \tan A$.

Express the values of the other functions in terms

25. Of $\sin A$.
26. Of $\cos A$.
27. Of $\tan A$.
28. Of $\cot A$.
29. Of $\sec A$.
30. Of $\csc A$.

31. Given $\sin A \cos A = .8$, to find $\sin A$ and $\cos A$.

32. Given $\sin A \cos A = \frac{1}{4}\sqrt{3}$, to find $\sin A$ and $\cos A$.

23. Given $\sin A (\sin A - \cos A) = \frac{4}{25}$, to find $\sin A$ and $\cos A$.

Trigonometrical Functions of Special Angles.

28. We will now show how to find the sine, cosine, etc. of some particular angles.

I. *The sine, cosine, etc. of an angle of 45°.*

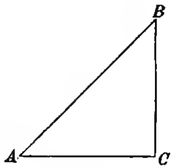

Fig. 4.

In the right triangle ABC, suppose the angle A equals 45°; then angle $B = 45°$ and $AC = BC$. Now,

$$\overline{AC}^2 + \overline{BC}^2 = \overline{AB}^2, \text{ or } 2\overline{BC}^2 = \overline{AB}^2.$$

Hence, $$\frac{\overline{BC}^2}{\overline{AB}^2} = \frac{1}{2}; \text{ and } \frac{BC}{AB} = \frac{1}{\sqrt{2}}.$$

Therefore,

$$\sin 45° = \frac{BC}{AB} = \frac{1}{\sqrt{2}}; \text{ and } \cos 45° = \frac{AC}{AB} = \frac{1}{\sqrt{2}}.$$ $= \frac{1}{1.414}$

Also, $$\tan 45° = \frac{BC}{AC} = 1; \text{ and } \cot 45° = \frac{AC}{BC} = 1.$$

And, $\sec 45° = \frac{AB}{AC} = \sqrt{2}$; and $\csc 45° = \frac{AB}{BC} = \sqrt{2}$.

$$\text{vers } 45° = 1 - \cos 45° = 1 - \frac{1}{\sqrt{2}}.$$

II. *The sine, cosine, tangent, etc. of an angle of 60°.*

In the right triangle ABC, let the angle $A = 60°$; then angle $B = 30°$. Produce AC to A' making $CA' = AC$. Then ABA' is an equilateral triangle; and $AB = A'B = AA'$; and $AC = \frac{1}{2} AA' = \frac{1}{2} AB$.

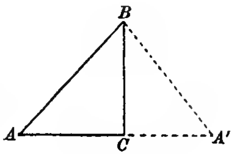

Fig. 5.

Then, $$\cos 60° = \frac{AC}{AB} = \frac{\frac{1}{2}AB}{AB} = \tfrac{1}{2}.$$

$$\sin 60° = \sqrt{1 - \cos^2 60°} = \sqrt{1 - \tfrac{1}{4}} = \frac{\sqrt{3}}{2}.$$

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}}{2} \div \tfrac{1}{2} = \sqrt{3}.$$

$$\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}.$$

$$\sec 60° = \frac{1}{\cos 60°} = 1 \div \tfrac{1}{2} = 2.$$

$$\csc 60° = \frac{1}{\sin 60°} = 1 \div \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}}.$$

$$\text{vers } 60° = 1 - \cos 60° = 1 - \tfrac{1}{2} = \tfrac{1}{2}.$$

III. *The sine, cosine, tangent, etc. of an angle of 30°.*

From Art. 26 and the previous solution,

$\sin 30° = \cos 60° = \frac{1}{2}$; $\cos 30° = \sin 60° = \frac{1}{2}\sqrt{3}$.

$\tan 30° = \cot 60° = \frac{1}{3}\sqrt{3}$; $\cot 30° = \tan 60° = \sqrt{3}$.

$\sec 30° = \csc 60° = \frac{2}{3}\sqrt{3}$; $\csc 30° = \sec 60° = 2$.

The Idea of Projections.

29. We introduced the trigonometrical functions as related to a right triangle. We now present a more general conception of the subject by the use of projections.

30. If a line is drawn through A, perpendicular to AC, and BD is drawn parallel to AC, then $AD = BC$ is the projection of AB on PQ, and AC is the projection of AB on AC. Calling the line AD or BC the *vertical* projection, and AC the *horizontal* projection, we can define the trigonometrical functions as follows:

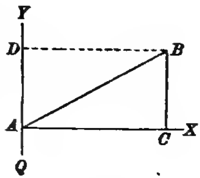

Fig. 6.

1. The *sine* is the ratio of the vertical projection of a line AB to the line AB.

2. The *cosine* is the ratio of the horizontal projection of a line AB to the line AB.

3. The *tangent* is the ratio of the vertical projection of a line AB to its horizontal projection.

4. The *cotangent* is the ratio of the horizontal projection of a line AB to its vertical projection.

EXERCISES IV.

1. Given $\tan A = \cot A$, to find angle A.

SOLUTION.—By Art. 26, $\cot A = 1 \div \tan A$; hence $\tan A = 1 \div \tan A$; hence $\tan^2 A = 1$, or $\tan A = 1$. Hence, Art. 28, $A = 45°$. Also infer by Art. 24.

Find A in the following:

2. Given $\sin A = \cos 2A$.
3. Given $\cos A = \sin 2A$.
4. Given $\tan 2A = \cot 2A$.
5. Given $\cos A = \sec A$.
6. Given $\sin 2A = \csc 2A$.
7. Given $\sin A = \cos 3A$.

SECTION III.

FUNCTIONS OF ANGLES IN GENERAL.

31. The definitions of sine, cosine, etc., given in Art. 23, apply only to acute angles. But the angles of triangles are often obtuse; hence it is necessary to take a more general view of angular magnitude and their functions.

32. If in the diagram we suppose OA to revolve from the position OA to OA_1, from right to left, in the direction of the arc AA_1, it will describe a right angle, or an angle of 90°. When OA arrives at OA_2, it will have described two right angles, or an angular magnitude of 180°; at A_3, three right angles, or 270°; at OA, four right angles, or 360°.

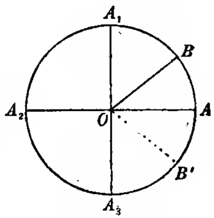

Fig. 7.

33. If the line OA should continue its revolution, when it arrives at OA_1 again it will have described five right angles, or 450°; and in this way we may conceive of an angular magnitude of any number of degrees. Similarly, we may have arcs of one or more circumferences.

34. It thus becomes necessary to extend the meaning of the trigonometrical functions and determine their values for different positions of the line AB (see Fig. 8); that is, for angles greater than 90°.

35. For this purpose all angles are estimated from the line AN (see Fig. 8), as follows:

1. Any angle, NAB, less than 90°, is said to be in the first quadrant.

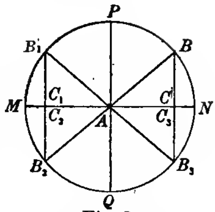

Fig. 8.

2. Any angle, NAB_1, greater than 90° and less than 180°, is said to be in the second quadrant.

3. Any angle, NAB_2, greater than 180° and less than 270°, is said to be in the third quadrant.

4. Any angle, NAB_3, greater than 270° and less than 360°, is said to be in the fourth quadrant.

36. Now, suppose the line AN to revolve successively to B, B_1, B_2, and B_3, forming figures like those below. Then generalizing the conception of sine, cosine, tangent, etc., as given in Art. 23, we have the following definitions of the trigonometrical functions for the four quadrants:

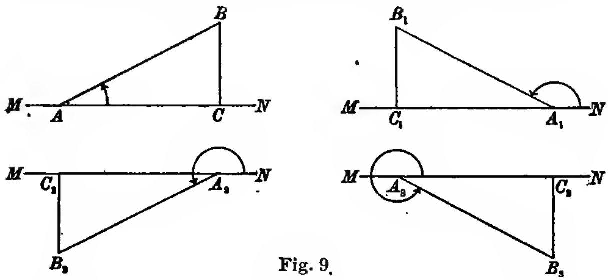

Fig. 9.

1. Denoting the angle NAB by A, we have, as already explained,

$$\sin A = \frac{BC}{AB}; \ \cos A = \frac{AC}{AB}; \ \tan A = \frac{BC}{AC}; \text{ etc.}$$

2. Denoting the angle NA_1B_1 by A_1, we have

$$\sin A_1 = \frac{B_1C_1}{A_1B_1}; \ \cos A_1 = \frac{AC_1}{A_1B_1}; \ \tan A_1 = \frac{B_1C_1}{A_1C_1}; \text{ etc.}$$

3. Denoting the angle NA_2B_2 by A_2, we have

$$\sin A_2 = \frac{B_2C_2}{A_2B_2}; \ \cos A_2 = \frac{AC_2}{A_2B_2}; \ \tan A_2 = \frac{B_2C_2}{A_2C_2}; \text{ etc.}$$

4. Denoting the angle NAB_3 by A_3, we have

$$\sin A_3 = \frac{B_3C_3}{AB}; \cos A_3 = \frac{AC_3}{AB}; \tan A_3 = \frac{B_3C_3}{AC_3}; \text{ etc.}$$

37. From this general conception of trigonometrical functions we may give the following general definitions to these functions :

1. The *sine* of an angle is the ratio of the vertical projection of the moving radius to the radius.

2. The *cosine* of an angle is the ratio of the horizontal projection of the moving radius to the radius.

3. The *tangent* of an angle is the ratio of the vertical projection of the moving radius to the horizontal projection.

4. The *cotangent* of an angle is the ratio of the horizontal projection to the vertical projection.

5. The *secant* of an angle is the reciprocal of the cosine.

6. The *cosecant* of an angle is the reciprocal of the sine.

NOTE.—If we call the horizontal projection of the moving radius the *abscissa*, and the vertical projection the *ordinate*, and the line AB the moving *radius*, we shall have the following simple general definitions of sine, cosine, etc. :

1. The sine of an angle is the ratio of the *ordinate* to the *radius*.

2. The cosine of an angle is the ratio of the *abscissa* to the *radius*.

3. The tangent of an angle is the ratio of the *ordinate* to the *abscissa*.

4. The cotangent of an angle is the ratio of the *abscissa* to the *ordinate*.

Algebraic Signs of Trigonometrical Functions.

38. In order to distinguish the trigonometrical functions of the different quadrants, it has been found convenient to use the signs *plus* and *minus* as we do in Algebra. The principles by which the signs of the functions are determined will now be explained.

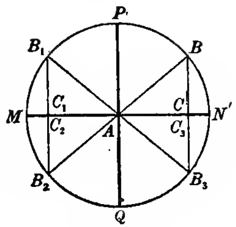

Fig. 10.

39. Suppose two lines, as MN and PQ, to intersect each other at right angles in the point A. Then,

1. *All lines estimated upward from MN are* POSITIVE; *and all lines estimated downward from MN are* NEGATIVE.

2. *All lines estimated from the vertical line PQ toward the right are* POSITIVE; *and all lines estimated from PQ toward the left are* NEGATIVE.

Thus, in Fig. 10, AC and BC are *plus*; B_1C_1 is *plus*; AC_1 is *minus*; B_2C_2 is *minus*; and B_3C_3 is *minus*. The sign of AB is not supposed to change for the different positions AB_1, AB_2, etc.

40. These principles, combined with the definitions of Art. 37, enable us to determine the algebraic sign of the trigonometrical functions of angles in the four quadrants. Thus, in the expressions of Art. 36:

1. Since BC and AC are both positive in the first quadrant, the sine, cosine, tangent, etc. of A are plus.

2. Since B_1C_1 is positive, and AC_1 is negative, in the second quadrant the sine is plus, the cosine minus, the tangent (= sine ÷ cosine) is minus; etc.

3. Since B_2C_2 and AC_2 are both negative in the third quadrant, the sine and cosine are both minus, the tangent (= sine ÷ cosine) is plus; etc.

4. Since B_3C_3 is negative and AC_3 is positive in the fourth quadrant, the sine is minus, the cosine plus, the tangent minus; etc.

41. If we let A, A_1, A_2, and A_3 denote respectively the angles CAB, CAB_1, etc., as in Art. 36, we have the following:

1. Sin A is $+$; cos A is $+$; tan A is $+$; cot A is $+$; sec A is $+$; etc.

2. Sin A_1 is $+$; cos A_1 is $-$; tan A_1 is $-$; cot A_1 is $-$; sec A_1 is $-$; etc.

3. Sin A_2 is $-$; cos A_2 is $-$; tan A_2 is $+$; cot A_2 is $+$; sec A_2 is $-$; etc.

4. Sin A_3 is $-$; cos A_3 is $+$; tan A_3 is $-$; cot A_3 is $-$; sec A_3 is $+$; etc.

42. These values of the trigonometrical functions in the different quadrants are concisely represented in the accompanying table:

Quadrants	I.	II.	III.	IV.
Sine and Cosecant	+	+	−	−
Cosine and Secant . . .	+	−	−	+
Tangent and Cotangent . . .	+	−	+	−

43. The following exercises will serve to fix these principles clearly in the mind. Let the student illustrate them with a diagram.

EXERCISES V.

Show the sign of each function of an angle,

1. Of 60°. 3. Of 100°. 5. Of 200°. 7. Of 300°.
2. Of 75°. 4. Of 150°. 6. Of 250°. 8. Of 320°.

9. In what quadrant is 127°? 256°? 295°? 470°? 510°? $\frac{2}{3}\pi$? $\frac{5}{4}\pi$? 3π? $\pi + 40°$? $\pi + 100$? $2\pi + 60°$? $n\pi + 45$?

10. Give the quadrant of angle A, if sin A is $+$, and tan A is $-$.

11. Give the quadrant of angle A, if cos A is $-$, and cot A is $+$.

12. Give the limits of angle A, if tan A is $-$, and csc A is $+$.

13. Give the limits of angle A, if tan A is $+$, and sec A is $-$.

14. If $\tan A = -\frac{3}{4}$, and $\cos A$ is negative, tell the quadrant of A, and find values of all the functions of A.

15. If $\sin A = -\frac{3}{5}$, and $\tan A$ is positive, tell the limits of A, and find the values of all the functions of A.

16. In a triangle, which functions may be negative, and when?

17. In a triangle, which functions will determine the angle, and which will not?

18. For what angle in each quadrant are the absolute values of the sine and cosine the same?

Functions of Negative Angles.

44. Angles may also be regarded as positive and negative when reckoned in opposite directions. Thus, in Fig. 7, if we regard angles reckoned from OA around in the direction of AA_1 as *positive*, angles reckoned in the opposite direction towards AA_3 may be regarded as *negative*.

45. Suppose, in Fig. 11, BCB_3 is perpendicular to CC_1, and $BC = B_3C$; then the sides and angles of the two triangles BAC and B_3AC are respectively equal. Let A denote angle CAB, then $-A$ will denote angle CAB_3.

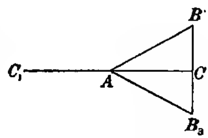

Fig. 11.

1. Now, $\sin A = \dfrac{BC}{AB}$; and $\sin.(-A) = \dfrac{B_3C}{AB_3}$;

and BC and B_3C are numerically equal, but have opposite signs (Art. 41); hence the results are equal with opposite signs.

Therefore, $\sin(-A) = -\sin A$.

2. Also, $\cos A = \dfrac{AC}{AB}$; and $\cos(-A) = \dfrac{AC}{AB_2}$;

and the lines are equal with the same signs.

Therefore, $\cos (= A) = \cos A.$

3. Also, $\tan (-A) = \dfrac{\sin (-A)}{\cos (-A)} = \dfrac{-\sin A}{\cos A} = -\tan A.$

4. Also, $\cot (-A) = \dfrac{\cos (-A)}{\sin (-A)} = \dfrac{\cos A}{-\sin A} = -\cot A.$

5. Also, $\sec (-A) = \dfrac{1}{\cos (-A)} = \dfrac{1}{\cos A} = \sec A.$

6. Also, $\csc (-A) = \dfrac{1}{\sin (-A)} = \dfrac{1}{-\sin A} = -\csc A.$

Extension of Fundamental Formulas.

46. The Fundamental Formulas of Art. 26 were derived for acute angles, but it may be readily shown that they apply to angles of any magnitude.

1. Thus, For. [1], Art. 26,

$$\sin^2 A + \cos^2 A = 1,$$

holds for all values of A; for whether a and b are plus or minus, a^2 and b^2 are always plus, and c^2 is plus; therefore the formula is always true.

2. So also For. [2], Art. 26,

$$\tan A = \frac{\sin A}{\cos A},$$

holds for all values of A; for both members equal $a \div b$ (see Art. 26), and hence will be equal whatever be the signs of a and b.

3. In the same manner all the formulas of Table I., Art. 27, are shown to be true for any value of angle A.

Note.—It will be interesting to the student to derive these formulas for angles terminating in each one of the four quadrants.

Reduction of Functions to the First Quadrant.

47. Having seen that the trigonometrical functions apply to angles of any magnitude, we shall now show that the functions of all angles greater than a right angle can be reduced to functions of angles less than a right angle.

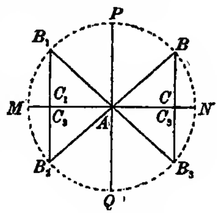

Fig. 12.

48. Suppose, in Fig. 12, that the diameters BB_2 and B_1B_3 are drawn, making equal angles with MN. Then the triangles BAC, B_1AC_1, B_2AC_2, and B_3AC_3 are equal. Denote the angle NAB by A.

Then angle $NAB_1 = 180° - A$.

1. Now, $\sin NAB_1 = \dfrac{B_1C_1}{AB_1}$; and $A = \dfrac{BC}{AB}$.

But BC and B_1C_1 are equal in magnitude and have the same sign.

Hence, $\sin(180° - A) = \sin A$.

2. Also, $\cos NAB_1 = \dfrac{AC_1}{AB_1}$; and $\cos A = \dfrac{AC}{AB}$:

Now, AC_1 and AC are equal in magnitude, but have opposite signs,

Hence, $\cos(180° - A) = -\cos A$.

49. The signs of the other trigonometrical functions may be found in a similar manner from the figure; but a simpler method is to use the results already obtained, as given in Art. 26. Thus,

3. $\tan(180° - A) = \dfrac{\sin(180° - A)}{\cos(180° - A)} = \dfrac{\sin A}{-\cos A} = -\tan A.$

4. $\cot(180° - A) = \dfrac{\cos(180° - A)}{\sin(180° - A)} = \dfrac{-\cos A}{\sin A} = -\cot A.$

5. $\sec(180° - A) = \dfrac{1}{\cos(180° - A)} = \dfrac{1}{-\cos A} = -\sec A.$

6. $\csc(180° - A) = \dfrac{1}{\sin(180° - A)} = \dfrac{1}{\sin A} = \csc A.$

7. $\text{vers}(180° - A) = 1 - \cos(180° - A) = 1 + \cos A.$

50. From Fig. 12, we see that the angle $NAB_2 = 180° + A$, and angle $NAB_3 = 360° - A$; hence we can find the trigonometrical functions of $180° + A$ and $360° - A$, as we found them for $180° - A$ in Arts. 48 and 49. These are given in Table II.

51. Again, in Fig. 13, suppose the radii so drawn that the angles NAB, PAB_1, QAB_2, and QAB_3 are all equal; then the triangles BAC, B_1AC_1, B_2AC_2, etc., are equal. Denote the angle NAB by A; then reckoning from N around toward the left,

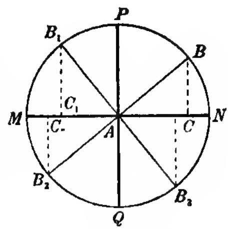

Fig. 13.

$$NAB_1 = 90° + A; \qquad NAB_2 = 270° - A.$$
$$NAB_3 = 270° + A.$$

1. Now, $\quad \sin NAB_1 = \dfrac{B_1C_1}{AB_1} = \dfrac{AC}{AB} = \cos A.$

Hence, $\quad \sin(90° + A) = \cos A.$

2. Also, $\cos NAB_1 = -\dfrac{AC_1}{AB_1} = -\dfrac{BC}{AB} = -\sin A.$

Hence, $\quad \cos(90° + A) = -\sin A.$

3. Also, $\quad \tan(90° + A) = -\cot A;$

and $\quad \cot(90° + A) = -\tan A.$

52. In a similar manner we may find the values of the functions of $270° - A$, and $270° + A$. All of these are embraced in the following table:

TABLE II.

Angle $= 90° + A$.		Angle $= 270° - A$.	
$\sin = \cos A$,	$\cot = -\tan A$,	$\sin = -\cos A$,	$\cot = \tan A$,
$\cos = -\sin A$,	$\sec = -\csc A$,	$\cos = -\sin A$,	$\sec = -\csc A$,
$\tan = -\cot A$,	$\csc = \sec A$.	$\tan = \cot A$,	$\csc = -\sec A$.
Angle $= 180° - A$.		**Angle $= 270° + A$.**	
$\sin = \sin A$,	$\cot = -\cot A$,	$\sin = -\cos A$,	$\cot = -\tan A$,
$\cos = -\cos A$,	$\sec = -\sec A$,	$\cos = \sin A$,	$\sec = \csc A$,
$\tan = -\tan A$,	$\csc = \csc A$.	$\tan = -\cot A$,	$\csc = -\sec A$.
Angle $= 180° + A$.		**Angle $= 360° - A$.**	
$\sin = -\sin A$,	$\cot = \cot A$,	$\sin = -\sin A$,	$\cot = -\cot A$,
$\cos = -\cos A$,	$\sec = -\sec A$,	$\cos = \cos A$,	$\sec = \sec A$,
$\tan = \tan A$,	$\csc = -\csc A$.	$\tan = -\tan A$,	$\csc = -\csc A$.

NOTE.—It will be well to have students derive all the values in the above table. These values can be easily remembered by observing that when the angle is connected with 180° or 360°, the functions in both columns have the *same name;* but when connected with 90° or 270°, they have *different names.*

53. From what has now been presented, we see that the trigonometrical functions of angles of any magnitude may be expressed in functions of angles less than 45°. The same is also readily shown to be true of negative angles.

Thus, $\sin 120° = \sin (90° + 30°) = \cos 30°$.

$\tan 223° = \tan (180° + 43°) = \tan 43°$.

$\cot 304° = \cot (270° + 34°) = -\tan 34°$.

54. The functions of $360° + x$, it is readily seen, are the same as those of x, since the moving radius has the same position in both cases. In general, if n denotes any positive whole number,

The functions of $(n \times 360° + x)$ *are the same as those of* x.

55. Hence, when the angle is greater than 360°, we may subtract 360° one or more times until we obtain an angle less than 360°; and the trigonometrical functions of this remainder will be the same as that of the given angle. This remainder being less than 360°, its functions can be expressed in functions of an angle less than 45°.

56. From the principle that the functions of all angles can be expressed in function of angles less than 45°, in the *tables of sines and cosines* we have only positive angles or arcs, and those of less than 45°.

EXERCISES VI.

1. Express the sine and cosine of 145° in functions of an angle less than 45°.

SOLUTION.—Sin $145° = \sin(180° - 35°) = \sin 35°$. Also, $\cos 145° = \cos(180° - 35°) = -\cos 35°$.

Express the following in functions of positive angles less than 45°:

2. Sin 170°.	9. Sec 246°.	16. Tan $\frac{3}{4}\pi$.
3. Cos 105°.	10. Csc 395°.	17. Cot $\frac{4}{5}\pi$.
4. Tan 125°.	11. Sin 412°.	18. Sin $(-60°)$.
5. Cot 204°.	12. Cos 846°.	19. Cos $(-130°)$.
6. Tan 300°.	13. Sin $(-35°)$.	20. Tan $(-200°)$.
7. Sin $(\pi + a)$.	14. Sin $(2\pi + a)$.	21. Cot $(-250°)$.
8. Cos $(\pi + a)$.	15. Cos $(2\pi - a)$.	22. Sec $(2n\pi + a)$.

23. Derive the following table of values:

Angle .	30°	45°	60°	120°	135°	150°	210°	225°
Sine . . .	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$
Cosine . .	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$
Tangent .	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
Cotangent	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
Secant . .	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	-2	$-\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
Cosecant .	2	$\sqrt{2}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	-2	-2

Extension of Formulas of Table II.

57. The formulas of Table II. were derived on the supposition that the angle A is less than 90°; but they are true whatever is the value of A.

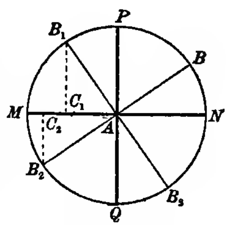

Fig. 14.

In order to prove this we will show first that they are true for $90° + A$, when A is obtuse. Let the angle NAB_1 be denoted by A. Draw BB_2 perpendicular to B_1B_3; then

$$\text{angle } NAB_2 = 90° + A.$$

Now, $\sin(90° + A) = -\frac{B_2C_2}{AB_2} = -\frac{AC_1}{AB_1} = \cos A.$

Hence, $\sin(90° + A) = \cos A.$

Similarly, it may be shown that

$$\cos(90° + A) = -\sin A.$$

These two formulas correspond with those of Table II., and the other formulas drawn from these will also correspond with those of the Table.

Hence all the formulas of $90° + A$, when A is obtuse, are the same as those when A is acute. Similarly, it can be shown that they are true when A terminates in the third or fourth quadrant. Therefore they are universally true.

58. In a similar manner it may be shown that all the other formulas of Table II. are true for any value of the angle A.

EXERCISES VII.

1. Express sine and cosine of 257° in functions of an angle less than 45°.

SOLUTION.— $\sin 257° = -\cos (270° - 257°) = -\cos 17°$.

NOTE.—The difference between this method of solution and that of Art. 56 will be readily seen.

Find the functions of the following in angles less than 45°:

2. 108°.	5. 196°.	8. 240°.	11. $-125°$.
3. 136°.	6. 215°.	9. 318°.	12. $-265°$.
4. $\pi - 30°$.	7. $\pi - \frac{2}{3}\pi$.	10. $2\pi - \frac{3}{4}\pi$.	13. $30° - 2\pi$.

Find the functions of the following in terms of the functions of x:

14. $x - 90°$.	17. $x - 360°$.	20. $x + 450°$.
15. $x - 180°$.	18. $x + 360°$.	21. $x - 540°$.
16. $x - 270°$.	19. $x - 450°$.	22. $x + 540°$.

Limiting Values of Trigonometrical Functions.

59. The *Limiting Values* of trigonometrical functions are their values at the beginning and end of the different quadrants.

These values are determined by the principle that *the*

value of a variable up to the limit is equal to its value at the limit.

60. In Art. 23 we have

$$\sin A = \frac{a}{c}; \quad \text{and} \quad \cos A = \frac{b}{c}.$$

Now, if $A = 0$, $a = 0$, and $b = c$.

Hence, $\sin 0 = \frac{0}{c} = 0$; and $\cos 0 = \frac{b}{c} = 1$.

61. In Art. 23 we have

$$\tan A = \frac{\sin A}{\cos A}; \quad \text{and} \quad \cot A = \frac{\cos A}{\sin A}.$$

Hence, $\tan 0 = \frac{0}{1} = 0$; and $\cot 0 = \frac{1}{0} = \infty$.

62. In Art. 53 we have

$\sin(90° + A) = \cos A$; and $\cos(90° + A) = -\sin A$.

Hence, supposing $A = 0°$, we have

$\sin 90° = \cos 0° = 1$; and $\cos 90° = -\sin 0 = -0$.

63. Proceeding in a similar manner, we find the limiting values of all the functions as expressed in the following table:

TABLE III.

Arc = 0.	Arc = 90°.	Arc = 180°.	Arc = 270°.	Arc = 360°.
sin = 0	sin = 1	sin = 0	sin = − 1	sin = − 0
cos = 1	cos = 0	cos = − 1	cos = − 0	cos = 1
tan = 0	tan = ∞	tan = − 0	tan = ∞	tan = − 0
cot = ∞	cot = 0	cot = − ∞	cot = 0	cot = − ∞
sec = 1	sec = ∞	sec = − 1	sec = − ∞	sec = 1
csc = ∞	csc = 1	csc = ∞	csc = − 1	csc = − ∞

64. From the principles now explained we can often determine the angle from the trigonometrical functions by inspection.

EXERCISES VIII.

1. Given $\sin^2 a - \cos^2 a = 0$, to find a.

SOLUTION.—Transposing, we have $\sin^2 a = \cos^2 a$; whence $\sin a = \cos a$; hence $a = 45°$ or $225°$.

Find the angle a in the following:

2. $\tan a = 1$.
3. $\sin a = 1$.
4. $\cos a = -1$.
5. $\sin^2 a + \cos^3 a = 0$.
6. $\tan a + \cot a = 0$.
7. $\cot a - 2 \cos a = 0$.
8. $3 \sin^2 a + 2 \cos^2 a = 3$.
9. $\cos a = -\frac{1}{2}$.
10. $\sec^2 a = 2$.
11. $\csc^2 a = \frac{4}{3}$.
12. $\sin^2 a = 3 \cos^2 a$.
13. $\sin a + \cos a = 1$.
14. $\sin^2 a - 2 \cos a + \frac{1}{4} = 0$.
15. $3 \sec^4 a + 8 = 10 \sec^2 a$.

Prove the following:

16. $\sin A = \tan A \cos A$.
17. $\tan A + \cot A = \sec A \csc A$.
18. $\tan A \sin A = \sec A - \cos A$.
19. $\cot A \cos A = \csc A - \sin A$.
20. $\dfrac{\sin A + \cos A}{\sec A + \csc A} = \sin A \cos A$.
21. $\dfrac{\sin A + \tan A}{\cot A + \csc A} = \sin A \tan A$.
22. $\dfrac{1 + \sin A}{1 + \cos A} \times \dfrac{1 + \sec A}{1 + \csc A} = \tan A$.
23. $\dfrac{\cot A}{\csc A} = \cos A$.
24. $\dfrac{\sin A}{\csc A} = \sin^2 A$.
25. $\dfrac{\csc A}{\cot A} = \sec A$.
26. $\sec^2 A + \tan^2 A = \sec^4 A - \tan^4 A$.

SECTION IV.

THE SUM AND DIFFERENCE OF TWO ANGLES.

65. We shall now find formulas for the trigonometrical functions of the sum and difference of two angles.

Let the angle AOB be denoted by A and the angle BOC by B; then the angle $AOC = A + B$.

On OC take any point C, draw $CD \perp$ to OA, $CN \perp$ to OB, $MN \perp$ to CD, and $NE \perp$ to OA. Then the angle CNM is the complement of MNO, or NOA; therefore angle $NCM =$ angle A.

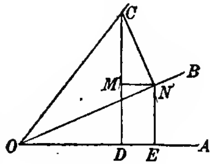

Fig. 15.

1. Now, $$CD = NE + CM.$$

Hence, $OC \sin(A + B) = ON \sin A + CN \cos A.$ Art. 23.

$$= OC \cos B \sin A + OC \sin B \cos A.$$

Whence $\quad \mathbf{\sin(A + B) = \sin A \cos B + \cos A \sin B.} \quad [9]$

2. Again, $$OD = OE - MN.$$

Hence, $OC \cos(A + B) = ON \cos A - NC \sin A.$

$$= OC \cos B \cos A - OC \sin B \sin A.$$

Whence, $\mathbf{\cos(A + B) = \cos A \cos B - \sin A \sin B.} \quad [10]$

66. These two formulas express the value of the sine and cosine of the sum of two angles in terms of the sines and cosines of the single angles. Enunciated in a theorem, the first gives

The sine of the sum of two angles is equal to the sine of the first into the cosine of the second, plus the cosine of the first into the sine of the second.

67. Again, in Fig 16, let the angle AOB be denoted by A, and the angle BOC by B; then angle NCM = angle $ENC = A$ (B. I. Th. 15).

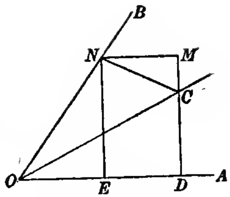

Fig. 16.

3. Now, $CD = NE - MC$.

Hence, $OC \sin(A - B) = ON \sin A - NC \cos A$.

$$= OC \cos B \sin A - OC \sin B \cos A.$$

Whence, $\mathbf{\sin(A - B) = \sin A \cos B - \cos A \sin B.}$ [11]

4. Again, $OD = OE + MN$.

Hence, $OC \cos(A - B) = ON \cos A + NC \sin A$.

$$= OC \cos B \cos A + OC \sin B \sin A.$$

Whence, $\mathbf{\cos(A - B) = \cos A \cos B + \sin A \sin B.}$ [12]

5. From Table I., For. 4, and formulas [9] and [10],

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}.$$

Dividing both terms of last member by $\cos A \cos B$, we have

$$\tan(A + B) = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}}.$$

Cancelling common factors, and reducing, we have

$$\mathbf{\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}. \qquad [13]$$

6. Substituting $-B$ for B in formula [13], and reducing, we have

$$\mathbf{\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}}. \qquad [14]$$

7. Dividing formula [10] by [9], and reducing as in [13], we have

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}. \qquad [15]$$

8. Substituting $-B$ for B in formula [15], and reducing, we have

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}. \qquad [16]$$

68. These eight formulas may be considered as the *Fundamental Theorems* of Trigonometry.

NOTE.—For [14] can be derived like [13], and [16] like [15].

Formulas for Double and Half Angles.

69. We now proceed to derive from these fundamental theorems the trigonometrical formulas for double and half angles.

1. Making $A = B$ in formulas [9], [10], [13], and [15], we have

$$\sin 2A = 2 \sin A \cos A \qquad [17]$$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad [18]$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad [19] \qquad \cot 2A = \frac{\cot^2 A - 1}{2 \cot A} \quad [20]$$

2. If in [18] we put $1 - \sin^2 A$ for $\cos^2 A$, and $1 - \cos^2 A$ for $\sin^2 A$, we have

$$\cos 2A = 1 - 2\sin^2 A. \qquad \cos 2A = 2\cos^2 A - 1.$$

Whence,

$$\sin A = \sqrt{\frac{1 - \cos 2A}{2}} \quad [21] \qquad \cos A = \sqrt{\frac{1 + \cos 2A}{2}} \quad [22]$$

Dividing [21] by [22], and then [22] by [21], multiplying numerator and denominator by the denominator, and reducing, we have

$$\tan A = \frac{\sin 2A}{1+\cos 2A} \quad [23] \qquad \cot A = \frac{\sin 2A}{1-\cos 2A} \quad [24]$$

3. Substituting $\frac{1}{2}A$ for A in [21], [22], [23], and [24], we have

$$\sin \tfrac{1}{2}A = \sqrt{\frac{1-\cos A}{2}} \quad [25] \qquad \cos \tfrac{1}{2}A = \sqrt{\frac{1+\cos A}{2}} \quad [26]$$

$$\tan \tfrac{1}{2}A = \frac{\sin A}{1+\cos A} \quad [27] \qquad \cot \tfrac{1}{2}A = \frac{\sin A}{1-\cos A} \quad [28]$$

Taking reciprocals of [27] and [28], we have

$$\cot \tfrac{1}{2}A = \frac{1+\cos A}{\sin A} \quad [29] \qquad \tan \tfrac{1}{2}A = \frac{1-\cos A}{\sin A} \quad [30]$$

70. Sums and Differences of Functions.

1. Adding and subtracting formulas [9] and [11], and doing the same with [10] and [12], we have

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B, \quad (1)$$
$$\sin(A+B) - \sin(A-B) = 2\cos A\sin B, \quad (2)$$
$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B, \quad (3)$$
$$\cos(A-B) - \cos(A+B) = 2\sin A\sin B. \quad (4)$$

2. Now, making

$$A + B = p \text{ and } A - B = q,$$

whence, $A = \frac{1}{2}(p+q)$ and $B = \frac{1}{2}(p-q)$;

and substituting these in the above, and we have,

$$\sin p + \sin q = 2\sin\tfrac{1}{2}(p+q)\cos\tfrac{1}{2}(p-q), \quad [31]$$
$$\sin p - \sin q = 2\cos\tfrac{1}{2}(p+q)\sin\tfrac{1}{2}(p-q), \quad [32]$$
$$\cos p + \cos q = 2\cos\tfrac{1}{2}(p+q)\cos\tfrac{1}{2}(p-q), \quad [33]$$
$$\cos q - \cos p = 2\sin\tfrac{1}{2}(p+q)\sin\tfrac{1}{2}(p-q). \quad [34]$$

3. Now dividing [31] by [32],

$$\frac{\sin p+\sin q}{\sin p-\sin q}=\frac{\sin \frac{1}{2}(p+q)\cos \frac{1}{2}(p-q)}{\cos \frac{1}{2}(p+q)\sin \frac{1}{2}(p-q)}=\frac{\tan \frac{1}{2}(p+q)}{\tan \frac{1}{2}(p-q)}. \quad [35]$$

In a similar manner, we obtain

$$\frac{\sin p+\sin q}{\cos p+\cos q}=\frac{2\sin \frac{1}{2}(p+q)\cos \frac{1}{2}(p-q)}{2\cos \frac{1}{2}(p+q)\cos \frac{1}{2}(p-q)}=\tan \tfrac{1}{2}(p+q), \quad [36]$$

$$\frac{\sin p-\sin q}{\cos p+\cos q}=\frac{2\sin \frac{1}{2}(p-q)\cos \frac{1}{2}(p+q)}{2\cos \frac{1}{2}(p+q)\cos \frac{1}{2}(p-q)}=\tan \tfrac{1}{2}(p-q), \quad [37]$$

$$\frac{\sin p+\sin q}{\sin (p+q)}=\frac{2\sin \frac{1}{2}(p+q)\cos \frac{1}{2}(p-q)}{2\sin \frac{1}{2}(p+q)\cos \frac{1}{2}(p+q)}=\frac{\cos \frac{1}{2}(p-q)}{\cos \frac{1}{2}(p+q)}, \quad [38]$$

$$\frac{\sin p-\sin q}{\sin (p+q)}=\frac{2\sin \frac{1}{2}(p-q)\cos \frac{1}{2}(p+q)}{2\sin \frac{1}{2}(p+q)\cos \frac{1}{2}(p+q)}=\frac{\sin \frac{1}{2}(p-q)}{\sin \frac{1}{2}(p+q)}, \quad [39]$$

$$\frac{\sin (p-q)}{\sin p-\sin q}=\frac{2\sin \frac{1}{2}(p-q)\cos \frac{1}{2}(p-q)}{2\sin \frac{1}{2}(p-q)\cos \frac{1}{2}(p+q)}=\frac{\cos \frac{1}{2}(p-q)}{\cos \frac{1}{2}(p+q)}. \quad [40]$$

71. These formulas may be enunciated in propositions; thus formula [35] gives

The sum of the sines of two arcs is to the difference of their sines as the tangent of one-half of the sum of the arcs is to the tangent of one-half of their difference.

Formulas for Two Angles Generalized.

72. In the demonstration of Arts. 65 and 67 both A and B, and also their sum, are assumed to be acute angles. These formulas, however, are entirely general, as may be readily seen.

1. If the sum $A+B$ is obtuse, A and B being acute, as in Fig. 17, the proof is the same as in Art 65, except that the sign of OD will be negative, as NM is greater than OE. The formulas for sin $(A+B)$ and cos $(A+B)$ are

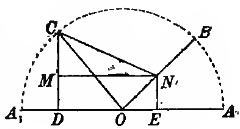

Fig. 17.

therefore true for all acute angles. Formulas [11] and [12] may be readily derived from formulas [9] and [10] by substituting $-B$ for B; hence these formulas are also true for all acute angles.

2. Again, let $AOB = A$ and $BOC = B$ be both obtuse angles. Draw $CN \perp$ to BC produced, $EN \perp$ to OA, $CD \perp$ to OA', and MN parallel to AA'. Then

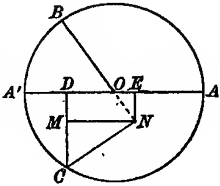

Fig. 18.

$CD = NE + CM$; whence, Art, 36,

$$OC \sin (A + B - 180°)$$
$$= ON \sin (180° - A) + CN \cos (180° - A)$$
$$= OC \cos (180° - B) \sin (180° - A)$$
$$+ OC \sin (180° - B) \cos (180° - A).$$

Whence, $\sin (A + B) = \sin A \cos B + \cos A \sin B.$

3. Again, considering the algebraic signs, we have

$$OD = OE - MN; \text{ whence}$$

$$OC \cos (A + B - 180°)$$
$$= ON \cos (180° - A) - NC \sin (180° - A)$$
$$= OC \cos (180° - B) \cos (180° - A)$$
$$- OC \sin (180° - B) \sin (180° - A).$$

Whence, $\cos (A + B) = \cos A \cos B - \sin A \sin B.$

Substituting $-B$ for B in each of the above formulas, we obtain $\sin (A - B)$ and $\cos (A - B)$, as in Art. 67. Hence, in the formulas of Arts. 65 and 67, and in all the formulas derived from them, A and B may be either acute or obtuse.

NOTE.—Another method of proving the universality of these formulas is given in the Supplement, Art. 149.

EXERCISES IX.

1. Given $\sin A = \frac{1}{2}$; find $\sin \frac{1}{2} A$; find $\cos \frac{1}{2} A$.
2. Given $\cos A = \frac{1}{2}$; find $\cos 2A$; find $\tan 2A$.
3. Given $\tan \frac{1}{2} A = 1$; find $\sin A$; find $\cos A$.
4. Given $\cot \frac{1}{2} A = \sqrt{2}$; find $\sin A$; find $\cos A$.

15. Find the trigonometrical function of an angle of 15°.

Solution.—Sin $15° = \sin (45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$. Substituting the values of sin 45°, cos 30°, etc., as given in Art. 56, and reducing, we have an expression for sin 15°. Similarly, we find all the values given below.

6. $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$.
7. $\cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.
8. $\tan 15° = 2 - \sqrt{3}$.
9. $\cot 15° = 2 + \sqrt{3}$.
10. $\sec 15° = \dfrac{2\sqrt{2}}{\sqrt{3}+1}$.
11. $\csc 15° = \dfrac{2\sqrt{2}}{\sqrt{3}-1}$.

Find sine, cosine, tangent, and cotangent of

12. 75°.
13. 105°.
14. 195°.
15. 240°.
16. $90° + A$.
17. $180° + A$.
18. $18° - A$.
19. $27° + A$.
20. $360° - A$.
21. $A - 180°$.
22. $450° + A$.
23. $30° - A$.

Other Formulas.

73. The student may now exercise his skill in demonstrating the following formulas. The Greek letter θ (*theta*) is used by many writers to denote any angle.

EXERCISES X.

Prove the following:

1. $\sin (30° + \theta) + \sin (30° - \theta) = \cos \theta$.

2. $\cos(60° + \theta) + \cos(60° - \theta) = \cos\theta$.

3. $\sin(60° + \theta) - \sin(60° - \theta) = \sin\theta$.

4. $\sin 31° + \sin 29° = \cos 1°$.

5. $\sin 62° - \sin 58° = \sin 2°$.

6. $\tan(45° + \theta) - \tan(45° - \theta) = 2\tan\theta$.

7. $\tan\theta + \cot\theta = 2\csc\theta$.

8. $\csc\theta - \cot\theta = \tan\frac{1}{2}\theta$.

9. $\csc\theta + \cot\theta = \cot\frac{1}{2}\theta$.

10. $\cot\frac{1}{2}\theta - \tan\frac{1}{2}\theta = 2\cot\theta$.

11. $\tan(\theta + 45°) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$.

12. $\tan(\theta - 45°) = \dfrac{\tan\theta - 1}{\tan\theta + 1}$.

13. $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos 2\theta$.

14. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

15. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

16. $\cos(\frac{2}{3}\pi + \theta) + \cos(\frac{2}{3}\pi - \theta) = -\cos\theta$.

17. $\cos 55° + \cos 65° + \cos 175° = 0$.

18. $\sin(n + 1)a + \sin(n - 1)a = 2\sin na \cos a$.

19. If $A + B + C = 180°$, prove $\tan A + \tan B + \tan C = \cot A \cot B \cot C$.

20. If $A + B + C = 90°$, prove $\cot A + \cot B + \cot C = \cot A \cot B \cot C$.

NOTE.—In 19th, $\tan(A + B) = \tan(180° - C)$; develop and simplify. Similarly, in 20th.

SECTION V.

THE THEOREMS OF TRIGONOMETRY.

74. The **Theorems of Trigonometry** express the relation between the sides and trigonometrical functions of the angles of a triangle.

75. These theorems are designed for the solution of triangles. By the solution of a triangle is meant the finding of the unknown parts from certain known parts.

Theorem I.

In any plane right triangle each side is equal to the product of the hypotenuse into the sine of the opposite angle.

Let ABC be a right triangle, right angled at C; then (Art. 23), we have

$$\sin A = \frac{a}{c}; \quad \text{and} \quad \sin B = \frac{b}{c}.$$

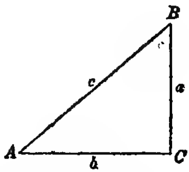

Fig. 19.

Hence, $a = c \sin A$; and $b = c \sin B$.

COR.—*In a plane right triangle each side is equal to the product of the hypotenuse into the cosine of the adjacent angle.*

Theorem II.

In any plane right triangle each side is equal to the product of the tangent of the opposite angle into the other side.

In the triangle ABC we have (Art. 23),

$$\tan A = \frac{a}{b}; \quad \text{and} \quad \tan B = \frac{b}{a}.$$

Hence, $\quad a = b \tan A; \quad$ and $\quad b = a \tan B.$

Cor.—*In a plane right triangle each side is equal to the product of the cotangent of the adjacent angle into the other side.*

Note.—These two theorems enable us to solve the different cases of right triangles.

Theorem III.

In any plane triangle the sides are proportional to the sines of the opposite angles.

Let ABC be a plane triangle whose angles are A, B, and C, and sides opposite these angles a, b, and c.

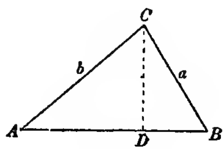

Fig. 20.

From C draw CD perpendicular to AB. Then in the right triangle ADC we have (Art. 23),

$$CD = AC \sin A \quad \text{and} \quad CD = BC \sin B.$$

Hence, $$AC \sin A = BC \sin B,$$

and $$AC : BC = \sin B : \sin A.$$

In a similar manner it may be shown that

$$AC : AB = \sin B : \sin C,$$

$$BC : AB = \sin A : \sin C.$$

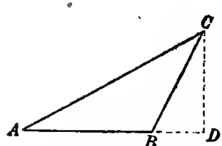

Fig. 21.

If the angle B is obtuse, as in Fig. 21, we have

$$CD = AC \sin A,$$

And $$CD = BC \sin (180° - B) = BC \sin B.$$

Hence, $$AC \sin A = BC \sin B.$$

Scholium.—This theorem enables us to solve a triangle when we have two angles and one side, or two sides and one angle not included by the sides.

Theorem IV.

In any plane triangle the sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.

Let ABC be any plane triangle whose angles are A, B, and C, and sides opposite these angles a, b, and c. Then, Th. III.,

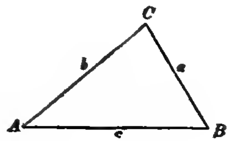

Fig. 22.

$$a : b = \sin A : \sin B.$$

Whence,

$$a + b : a - b = \sin A + \sin B : \sin A - \sin B.$$

But, $$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}. \qquad [35]$$

Hence, $a + b : a - b = \tan \frac{1}{2}(A + B) : \tan \frac{1}{2}(A - B)$.

Scholium.—This theorem enables us to solve a triangle when we have two sides and the included angle.

Theorem V.

In any plane triangle, if a line is drawn from the vertical angle perpendicular to the base, then the whole base will be to the sum of the other two sides as the difference of those sides is to the difference of the segments of the base.

Let ABC be any plane triangle, and CD a line drawn perpendicular to the base.

Then Th. XI., Book IV.,

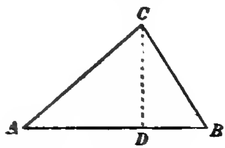

Fig. 23.

$$\overline{AC}^2 = \overline{AD}^2 + \overline{DC}^2,$$

and $$\overline{BC}^2 = \overline{BD}^2 + \overline{DC}^2.$$

Subtracting, $$\overline{AC}^2 - \overline{BC}^2 = \overline{AD}^2 - \overline{BD}^2.$$

Hence (B. IV., Th. X., C.),

$$(AC + BC)(AC - BC) = (AD + BD)(AD - BD).$$

Whence, $AD + DB : AC + BC = AC - BC : AD - DB.$

SCHOLIUM.—This theorem enables us to solve a triangle when the three sides are given.

Theorem VI.

In any plane triangle the square of any side is equal to the sum of the squares of the other two sides, diminished by twice the product of the two sides and the cosine of the included angle.

Let ABC be any plane triangle, and CD a line perpendicular to the base.

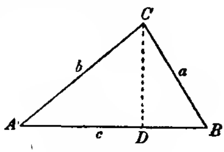

Fig. 24.

Then Th. IV., Book 13.,

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2AB \times BD;$$

Also, $BD = a \cos B$ (Th. I. C.).

Hence, $$b^2 = c^2 + a^2 - 2ac \cos B.$$

COR.—From Th. VI.,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}.$$

Similarly, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$; $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

SCHOLIUM.—These formulas can also be used to find the angles of a triangle when the three sides are given.

76. The formulas of Theorem VI. may be put in a more convenient form.

Now, $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}. \qquad \text{Th. VI.}$$

Also, $$1 - \cos A = 2\sin^2 \tfrac{1}{2} A. \qquad [25]$$

Whence, $$2\sin^2 \tfrac{1}{2} A = 1 - \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{(a + b - c)(a - b + c)}{2bc}.$$

Let $$2s = a + b + c;$$

Then, $a + b - c = 2(s - c)$ and $a - b + c = 2(s - b)$.

$$\left.\begin{aligned} &\text{Whence,} && \sin^2 \tfrac{1}{2} A = \frac{(s - b)(s - c)}{bc}. \\ &\text{Similarly,} && \cos^2 \tfrac{1}{2} A = \frac{s(s - a)}{bc}. \\ &\text{Hence,} && \tan^2 \tfrac{1}{2} A = \frac{(s - b)(s - c)}{s(s - a)}. \end{aligned}\right\} \quad [41]$$

By changing the letters we have

$$\left.\begin{aligned} \sin^2 \tfrac{1}{2} B &= \frac{(s - a)(s - c)}{ac}; & \sin^2 \tfrac{1}{2} C &= \frac{(s - a)(s - b)}{ab} \\ \cos^2 \tfrac{1}{2} B &= \frac{s(s - b)}{ac}; & \cos^2 \tfrac{1}{2} C &= \frac{s(s - c)}{ab}. \\ \tan^2 \tfrac{1}{2} B &= \frac{(s - a)(s - c)}{s(s - b)}; & \tan^2 \tfrac{1}{2} C &= \frac{(s - a)(s - b)}{s(s - c)} \end{aligned}\right\} \quad [42]$$

SECTION VI.

NUMERICAL VALUE OF SINES, TANGENTS, ETC.

77. The theorems now presented show the relation between the sides of a triangle and the trigonometrical functions of its angles. These sides are expressed in numbers; hence, to solve a triangle we must find the numerical value of these trigonometrical functions for any given angle.

78. When the radius of the circle is unity, the sine of the angle NAB (Fig. 8) is equal to the straight line BC.

When the angle is very small, the line BC is very nearly equal to the arc AB; hence the sine of a very small angle is very nearly equal to the arc which measures the angle.

By dividing $\pi = 3.1415926$ by the number of minutes in $180°$, we find the length of an arc of $1'$ to be 0.0002908882. This arc is so small that it does not differ materially from the sine of the angle of which it is the measure; hence, we may assume

$$\sin 1' = 0.0002908882.$$

We then find the cosine of $1'$ by For. [3], Table I.

Thus, $\quad \cos 1' = \sqrt{1 - \sin^2 1'} = .9999999577$, etc.

79. To find the sine of the other arcs, we take the formula under Art. 70, putting it in the form

$$\sin (a + b) = 2 \sin a \cos b - \sin (a - b).$$

Now, make $b = 1'$, and then in succession, a equal to $1'$, $2'$, $3'$, etc. and we have

$$\sin 2' = 2 \sin 1' \cos 1' - \sin 0 = 0.0005817764.$$
$$\sin 3' = 2 \sin 2' \cos 1' - \sin 1' = 0.0008726646.$$
$$\sin 4' = 2 \sin 3' \cos 1' - \sin 2' = 0.0011635526.$$
$$\sin 5' = \text{etc.}$$

Substituting in a similar manner in the formula

$$\cos (a + b) = 2 \cos a \cos b - \cos (a - b).$$

We find

$$\cos 2' = 2 \cos 1' \cos 1' - \cos 0' = 0.9999998308.$$
$$\cos 3' = 2 \cos 1' \cos 2' - \cos 1' = 0.9999996193.$$
$$\cos 4' = 2 \cos 1' \cos 3' - \cos 2' = 0.9999993232.$$
$$\text{etc.} \qquad\qquad\qquad \text{etc.}$$

80. We may thus obtain the sines and cosines of angles of any number of degrees and minutes up to 45°. Then,

since the sine and cosine of an angle are equal respectively to the cosine and sine of its complement, the sines and cosines of angles between 45° and 90° are immediately derived from those between 0° and 45°.

81. The tangents and cotangents may be found from the sines and cosines by the formulas,

$$\tan a = \frac{\sin a}{\cos a}; \quad \text{and} \quad \cot a = \frac{\cos a}{\sin a};$$

and the secants and cosecants by the formulas,

$$\sec a = \frac{1}{\cos a}; \quad \text{and} \quad \csc a = \frac{1}{\sin a}.$$

82. These numerical values of the sines, cosines, tangents, etc. of angles from 0° to 45°, arranged in a table, constitute what is called a TABLE OF NATURAL SINES, COSINES, etc.

NOTES.—1. In actual practice it is not necessary to continue the process of computation beyond 30°; for by Art. 70 we have, reducing,

$$\sin (30° + a) = \cos a - \sin (30° - a),$$
$$\cos (30° + a) = \cos (30° - a) - \sin a;$$

so that the table may be continued above 30° by simply subtracting the sines and cosines under 30° previously found.

2. The values of the sines, cosines, etc. thus computed are very nearly but not absolutely correct. The equation, arc $a = \sin a = \tan a$, is true for the natural functions of 30° as far as six decimal places, and for 1° as far as five decimal places. For any arc a it has been shown that $\sin a$ lies between a and $a - \frac{1}{4}a^3$; the values found above for large angles must therefore be corrected.

3. The results can be verified and corrected by means of independent calculations. Thus, $\cos 45° = \sqrt{\frac{1}{2}}$, Art 42; from which, by For. 22 and 23, we can find sine and cosine of 22° 30′, 11° 15′, etc. So also from $\cos 30° = \frac{1}{2}\sqrt{3}$, we can find sine and cosine of 15°, 7° 30′, 3° 45′, etc.

5

83. By means of these natural signs the sides and angles of triangles can be readily determined. Thus, suppose in the triangle ABC, page 60, we have given $a = 100$ ft., angle $A = 45°$, and angle $C = 60°$, to find b.

By Th. III., $\sin A : \sin B = a : b.$

Whence, $$b = \frac{a \sin B}{\sin A}.$$

Now, $a = 100$; $\sin B = \sin 60° = \frac{1}{2}\sqrt{3}$; $\sin A = \sin 45° = 1.$

Hence, $b = 100 \times \frac{1}{2}\sqrt{3} = 50\sqrt{3}.$

84. In this example the numbers are small and the calculation easily made. In general, however, the sines, cosines, etc. are expressed in large decimals, and the calculation is exceedingly tedious. To avoid this labor, logarithmic sines, cosines, etc. are used, which we shall now explain.

Logarithmic Sines, Cosines, Tangents, etc.

85. A **Logarithmic Sine, Cosine, Tangent** or **Cotangent** is the logarithm of the natural sine, cosine, tangent or cotangent.

86. The logarithmic sine, cosine, etc. of an angle is readily computed from the natural sine, cosine, etc., as follows:

1. We first find the logarithm of the natural sine or cosine. Then, since the sines and cosines of angles are less than unity, their logarithms would have *negative characteristics*. In order to avoid these negative quantities, it has been found convenient to increase the logarithm by 10, so we make the characteristic 9 instead of -1, 8 instead of -2, etc.

2. The tangents of angles under 45° are also less than unity, and

the characteristics of logarithmic tangents are also increased by 10. The same principle applies to logarithmic cotangents, secants, etc.

87. In using these logarithmic functions, therefore, we have the rule that *for each logarithmic function added in forming a sum, we must deduct* 10 *from that sum.*

88. The logarithmic tangent and cotangent are readily derived from the logarithmic sine and cosine by subtracting the one from the other.

Thus, $$\tan A = \frac{\sin A}{\cos A}.$$

Hence, $$\log \tan A = \log \sin A - \log \cos A.$$

Similarly, $$\log \cot A = \log \cos A - \log \sin A.$$

EXERCISES XI.

1. Given $\sin 36° 24' = .59342$, find log sin. *Ans.* 9.773361.
2. Given $\cos 64° 30' = .43051$, find log cos. *Ans.* 9.633984.
3. Given $\log \cos 65° 24' = 9.619386$, find cosine. *Ans.* .41628.
4. Given $\log \tan 59° 44' = 10.233905$, find tangent. *Ans.* .8639.
5. Find $\log \cos 36° 24'$ from Ex. 1. *Ans.* 9.905739.
6. Find $\log \tan 36° 24'$ from Ex. 1 and 5. *Ans.* 9.867622.

Logarithmic Tables and their Use.

89. **A Table of Logarithmic Sines,** etc. is a table containing the logarithmic sine, cosine, tangent and cotangent of angles, increased by 10. (See Appendix, p. 17.)

90. In the Table the degrees are given at the top and bottom of the page, and the minutes at the sides, in the column headed M.

91. The column headed D contains the *increase* or *decrease* for 1 second. This difference is found by subtracting the logarithmic sine, cosine, etc. of any angle from that

of the angle next exceeding it by 1 minute, and dividing the result by 60.

NOTE.—This use of the difference is based on the *principle of proportional parts*, which though not rigidly correct is nearly enough so for practical purposes.

92. We shall now explain the method of using the Tables of Logarithmic Functions.

93. *To find the logarithmic sines, cosines, etc. of angles or arcs.*

1. *When the angle is expressed in degrees, or in degrees and minutes.* If the *angle is less* than 45°, look for the degrees at the *top* of the page, and for the minutes in the *left*-hand column; then, opposite to the minutes, on the same horizontal line, in the columns headed *Sine*, will be found the logarithmic sine; in that headed *Cosine* will be found the logarithmic cosine, etc. Thus,

log sin 23° 35′	9.602150
log tan 23° 35′	9.640027

If the angle exceeds 45°, look for the degrees at the bottom of the page, and for the minutes in the *right*-hand column; then, opposite to the minutes, in the same horizontal line, in the column marked at the bottom *Sine*, will be found the logarithmic sine, etc. Thus,

log cos 65° 24′	9.619386
log tan 65° 24′	10.339290

2. *When the angle contains seconds.*—Find the logarithmic sine, etc. as before; then multiply the corresponding number found in column D by the number of seconds, and *add* the product to the preceding logarithm for the sines or tangents, and *subtract* it for cosines or cotangents.

We subtract for cosine and cotangent, because the greater the angle the less the cosine or cotangent. In multiplying the tabular difference by the number of seconds, we observe the same rule for the decimal point as in logarithms. If the angle is greater than 90°, we find the sine, cosine, etc. of its supplement.

EXERCISES XII.

1. Find the logarithmic sine of 36° 24′ 42″.

SOLUTION.

log sin 36° 24′,		9.773361
Tabular difference,	2.85	
No. of seconds,	42	
Product,	119.70 to be added,	120
log sin 36° 24′ 42″,		9.773481

2. Find the logarithmic cosine of 64° 30′ 30″.

SOLUTION.

log cos 64° 30′,		9.633984
Tabular difference,	4.41	
No. of seconds,	30	
Product	132.30 to be subtracted,	132
log cos 64° 30′ 30″,		9.633852

3. Find the logarithmic tangent of 120° 15′ 24″.

SOLUTION.

		180° 00′ 00″	
The given angle,		120° 15′ 24″	
Supplement,		59° 44′ 36″	
log tan 59° 44′,			10.233905
Tabular difference,	4.84		
No. of seconds,	36		
Product,	174.24, to be added		174
log tan 120° 15′ 24″,			10.234079

4. Find the log sine of 40° 40′ 40″. *Ans.* 9.814117.
5. Find the log cos of 140° 30′ 20″. *Ans.* 9.887441.
6. Find the log tan of 85° 25′ 45″. *Ans.* 11.097200.
7. Find the log cot of 144° 44′ 28″. *Ans.* 10.150603.

94. *To find the angle corresponding to any logarithmic sine, cosine, tangent, or cotangent.*

1. Look in the proper column of the table for the given logarithm; if found there, and the name of the function is at the *head* of the column, take the degrees at the *top*, and the minutes on the *left;* but

if the name of the function is at the *foot* of the column, take the degrees at the *bottom*, and the minutes on the *right*.

2. If the given logarithm is not exactly given in the table, then take the next less logarithm, subtract it from the given logarithm, and divide the remainder by the corresponding tabular difference; the quotient will be seconds, which must be *added* to the degrees and minutes corresponding to the logarithm taken from the table, for *sines* and *tangents*, and *subtracted* for *cosines* and *cotangents*.

EXERCISES XIII.

1. Find the angle whose logarithmic sine is 9.617033.

SOLUTION.

Given log sine,		9.617033
Next less in table,		9.616894
Tabular difference,	4.63)	139.00(30, to be added.

Hence the angle is 24° 27′ 30″.

2. Find the angle whose logarithmic cosine is 9.704682.

SOLUTION.

Given log cosine,		9.704682
Next less in table,		9.704610
Tabular difference,	3.58)	72.00(20, to be subtracted.

Hence, the angle is 59° 33′ 40″.

3. Find the angle whose log sine is 9.438672. *Ans.* 15° 56′ 14″.
4. Whose log cosine is 9.634520. *Ans.* 64° 27′ 47″.
5. Whose log tangent is 10.753246. *Ans.* 79° 59′ 24″.
6. Whose log cotangent is 11.449852. *Ans.* 2° 1′ 40″.

95. The secants and cosecants are omitted in the table, since they are easily derived from the sines and cosines. Thus, by Art. 26,

$$\sec A = \frac{1}{\cos A}, \qquad \text{and} \qquad \csc A = \frac{1}{\sin A}.$$

Whence, $\quad \sec A \cos A = 1; \quad$ and $\quad \csc A \sin A = 1.$

Taking the logarithm and observing to add 10 to each logarithm, we have

$$\log \sec A = 20 - \log \cos A.$$
$$\log \csc A = 20 - \log \sin A.$$

Hence, *the logarithmic secant is found by subtracting the logarithmic cosine from* 20, *and the logarithmic cosecant is found by subtracting the logarithmic sine from* 20.

EXERCISES XIV.

1. Find the log csc of $24° 27' 34''$. *Ans.* 10.382949.

2. Find the log sec of $54° 12' 40''$. *Ans.* 10.232992.

3. Prove that the log cot of an angle equals 20 minus the log tan of the angle, and conversely.

NOTES.—1. The sine of an angle near 90° varies much more slowly than the sine of an angle near 0°, while the opposite is true of their cosines. Hence, in finding an angle near 90° it is better to avoid the use of its sine, and in finding an angle near 0° it is better to avoid the use of its cosine. The tangent varies with the arc more rapidly than either its sine or cosine, and may be used equally well with any angle.

2. The Tables of Logarithmic sines, cosines, etc. extend to six decimal places. They can be easily changed in use to five-place logarithms by omitting the sixth decimal and adding one to the fifth decimal when the figure omitted is greater than 5. Thus, for log tan $23° 35' = 9.640027$, we may write log tan $23° 35' = 9.64003$. In a similar way six-place logarithms may be reduced to four-place and three-place logarithms.

Some mathematicians prefer five-place tables, and for work not requiring great accuracy even four-place and three-place tables are used.

SECTION VII.

THE SOLUTION OF TRIANGLES.

96. The Solution of Triangles is the process of finding the unknown parts when a sufficient number of the parts are given.

97. There are *six* parts in a plane triangle, and three of these, one of the three being a side, must be given to find the other parts.

98. If the angles alone were given, it is clear that the sides could not be determined, since there could be an indefinite number of triangles having their angles respectively equal.

Solution of Plane Right Triangles.

99. In the solution of right triangles we have the four following cases:

1 When the hypotenuse and one acute angle are given.
2. When the hypotenuse and a leg are given.
3. When one leg and either acute angle are given.
4. When the two legs are given.

Case I.

100. *Given the hypotenuse c and one acute angle A, to find the other parts.*

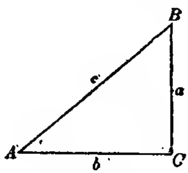

Fig. 25.

Method.—Let ABC denote the triangle. Then, to find a, we have, Th. I.,

$$\sin A = \frac{a}{c}.$$

Whence, $\log a = \log c + \log \sin A$.

Hence, to find a, we add $\log c$ to $\log \sin A$, and find the number corresponding to the resulting logarithm.

Similarly we find b; and $B = 90° - A$.

EXERCISES XV.

1. In a right triangle ABC, given the hypotenuse $c = 475$, and angle $A = 36° 34'$; find the remaining parts.

SOLUTION.—From the method given above we have the following operation:

$\log c\ (475) = 2.676694$	$\log c\ (475) = 2.676694$
$\log \sin A\ (36° 34') = 9.775070$	$\log \sin B\ (53° 26') = 9.904804$
$\log a = 2.451764$	$\log b = 2.581498$
$a = 282.985$	$b = 381.503$

NOTE.—In adding $\log \sin A$ to $\log c$, 10 is rejected from the sum to correct for the 10 which was added to the log. of the sine (Art. 87).

2. Given the hypotenuse $c = 45.36$, $A = 45° 36'$; find $a = 32.408$, $b = 31.736$, and $B = 44° 24'$.

3. Given $c = 250$, and $B = 37° 30'$; find $A = 52° 30'$, $a = 198.338$, and $b = 152.19$.

4. Given $c = 251.4$, $A = 75° 12'$; find $B = 14° 48'$, $a = 243.06$, and $b = 64.22$.

CASE II.

101. *Given the hypotenuse c and one of the legs a, to find the remaining parts.*

METHOD.—Let ABC denote the triangle. Then, to find the angle A,

We have $\sin A = \dfrac{a}{c}$. Th. I.

Whence, $\log \sin A = \log a - \log c$.

Similarly, $\log b = \log \sin B + \log c$.

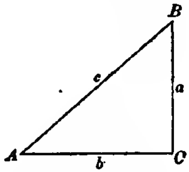

Fig. 26.

From these A and b are readily found. And $B = 90° - A$.

EXERCISES XVI.

1. Given the hypotenuse $c = 125$, and the side $a = 76.095$; to find the remaining parts.

SOLUTION.—From the method indicated above we have the following operations:

$\log a\ (76.095) = 1.881357$	$\log c\ (125) = 2.096910$
$\log c\ (125) = 2.096910$	$\log \sin B\ (52° 30°) = 9.899467$
$\log \sin A = 9.784447$	$\log b = 1.996377$
$A = 37° 30'$	$b = 99.169$
$B = 52° 30'$	

NOTE.—After subtracting it is necessary to add 10 to the result to give log sin A. In practice we add 10 to the miuuend before subtracting.

2. In a right triangle ABC, given $c = 400$, and $a = 240$; find $b = 320$, $A = 53° 7' 49''$, and $B = 36° 52' 11''$.

3. In a right triangle ABC, given $c = 396$, and $b = 218$; find $A = 56° 35' 54''$, $B = 33° 24' 6''$, and $a = 330.59$.

4. In a right triangle ABC, given $c = 126.206$, and $b = 97.72$; find $a = 82.507$, $A = 40° 10' 30''$, and $B = 49° 49' 30''$.

CASE III.

102. *Given one leg, as b, and either acute angle, as A, to find the remaining parts.*

METHOD.—From Th. II., we have

$$\tan A = \frac{a}{b};$$

Whence, $\log a = \log \tan A + \log b$.

Also, $\log c = \log a - \log \sin A$.

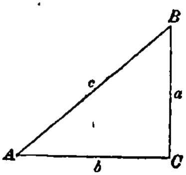

Fig. 27.

EXERCISES XVII.

1. In a right triangle ABC, given the side $b = 200$, and the angle $A = 34° 45'$; to find the other parts.

SOLUTION.—From the method indicated above, we have the following operations:

$\log \tan A\ (34° 45') = 9.841187$	$\log a\ (138.74) = 2.142217$
$\log b\ (200) = \underline{2.301030}$	$\log \sin A\ (34° 45') = \underline{9.755872}$
$\log a = 2.142217$	$\log c = 2.386345$
$a = 138.74$	$c = 243.41$

2. In a right triangle ABC, given $a = 364.3$, $A = 50° 45'$; find $b = 297.645$, $c = 470.433$, and $B = 39° 15'$.

3. In a right triangle ABC, given $b = 90.5$, and $A = 50° 30'$; find $a = 109.78$, $c = 142.27$, and $B = 39° 30'$.

4. In a right triangle, given $a = 305.34$, and $B = 50° 18' 32''$; find $b = 367.9$, $c = 478.1$, and $A = 39° 41' 28''$.

CASE IV.

103. *Given the two sides, a and b, about the right angle, to find the remaining parts.*

METHOD.—We have $\tan A = \frac{a}{b}$, Th. II.

Whence, $\log \tan A = \log a - \log b$.

Also, $\log c = \log a - \log \sin A$,

EXERCISES XVIII.

1. In a right triangle, the side $a = 239$, side $b = 188$; find the angles and hypotenuse.

SOLUTION.

$\log \tan A = \log a - \log b$	$\log c = \log a - \log \sin A$
$\log a\ (239) = 2.378398$	$\log a\ (239) = 2.378398$
$\log b\ (188) = \underline{2.274158}$	$\log \sin A\ (51° 48', \text{etc.}) = \underline{9.895409}$
$\log \tan A = 10.104240$	$\log c = 2.482989$
$A = 51° 48' 40''$	$c = 304.08$
$B = 38° 11' 20''$	

2. In a right triangle, given $a = 99.98$, $b = 152.71$; find $c = 182.5$, $A = 33° 12'$, $B = 56° 48'$.

3. In a right triangle, given $a = 515$, $b = 505$; find $A = 45° 33' 42''$, $B = 44° 26' 18''$, and $c = 721.28$.

4. In a right triangle, given $a = 29.37$, $b = 37.29$; find $c = 47.467$, $A = 38° 13' 28''$, $B = 51° 46' 32''$.

Solution of Plane Oblique Triangles.

104. In the solution of oblique triangles there are four cases, as follows: Given

1. Two angles and a side.
2. Two sides and an angle opposite to one of them.
3. Two sides and the included angle.
4. The three sides.

NOTE.—In the solution, let A, B, and C denote the angles of the triangle, and a, b, c denote the sides opposite these angles respectively.

CASE I.

105. *Given two angles A and B and one side a, to find the remaining parts.*

METHOD.—Let ABC be the triangle.

1. Then to find C, subtract the sum of A and B from 180°.

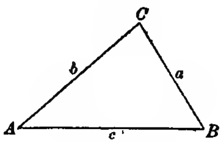

Fig. 28.

2. To find b we have (Th. III.)

$$a : b = \sin A : \sin B,$$

Whence, $$b = \frac{a \sin B}{\sin A}.$$

3. To find c we have

$$a : c = \sin A : \sin C,$$

Whence, $$c = \frac{a \sin C}{\sin A}.$$

EXERCISES XIX.

1. In the triangle ABC, given $A = 32° \ 24'$, $B = 40° \ 32'$, $a = 240$; find the remaining parts.

SOLUTION.—Applying the logarithms to the formulas given above, and substituting the numerical values, we have the following operations:

$a = 240$	$\log a = 2.380211$	$\log a = 2.380211$
$A = 32° \ 24'$	$\log \sin B = 9.812840$	$\log \sin C = 9.980442$
$B = 40° \ 32'$	$\text{colog} \sin A = 0.270976$	$\text{colog} \sin A = 0.270976$
$A + B = 72° \ 56'$	$\log b = 2.464027$	$\log c = 2.631629$
$C = 107° \ 04'$	$b = 291.09$	$c = 428.182$

2. In the triangle ABC, given $A = 27° \ 40'$, $C = 65° \ 45'$, $c = 625$; find $B = 86° \ 35'$, $a = 318.29$, $b = 684.266$.

3. In $\triangle \ ABC$, given $A = 30° \ 20'$, $B = 50° \ 10'$, and $c = 186.74$; find $C = 99° \ 30'$, $a = 95.62$, and $b = 145.39$.

4. In $\triangle \ ABC$, given $B = 51° \ 15' \ 35''$, $C = 37° \ 21', 25''$, and $a = 305.296$; find $A = 91° \ 23'$, $b = 238.197$, $c = 185.3$.

CASE II.

106. *Given two sides a and b, and the angle A opposite to the side a, to find the remaining parts.*

METHOD.—In this case we proceed as follows:

1. To find B, we have (Th. II.),

$$a : b = \sin A : \sin B; \quad \text{whence,} \quad \sin B = \frac{b \sin A}{a}.$$

2. To find C, we have, $\quad C = 180° - (A + B)$.

3. To find c, we have, Th. III.,

$$a : c = \sin A : \sin c; \quad \text{whence,} \quad c = \frac{a \sin C}{\sin A}.$$

Discussion.—Here the angle B is determined from its sine; and since the sine of an angle equals the sine of its supplement, the

angle B admits of two values, supplements of each other. We must therefore examine the problem to see which of the two angles (or if both) must be taken.

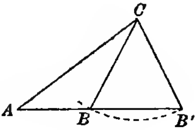

Fig 29.

Let ABC denote the triangle; then from the principles of Geometry we have the following conclusions:

1. If $a > b$, then $A > B$, and B must be acute; hence there is only *one value* of B, and *one*, and *only one*, triangle that will satisfy the given conditions.

2. If $a = b$, then $A = B$, and both A and B are acute, since their sum is less than 180°, and there is only *one value* of B, and *only one triangle*, and that is *isosceles.*

3. If $a < b$, then $A < B$; and A must be acute in order that the triangle is possible; and if A is acute, it is evident that there are *two triangles* ABC and $AB'C$ which satisfy the given conditions. The angles ABC and $AB'C$ are supplementary; hence in this case in finding B from $\sin B$, we use both the angle given by the table and its supplement.

4. In the formula, $\sin B = b \sin A \div a$, if $a = b \sin A$, then $\sin B = 1$; hence $B = 90°$, and the required triangle is a *right triangle.*

5. If $a < b \sin A$, then $\sin B > 1$, which is impossible, and the *triangle is impossible.* So also if $a < b$ and $A = 90°$.

NOTES.—1. In practice the number of solutions can be usually determined by the circumstances of the problem. When there is any doubt, compute the value of $b \sin A$, and compare it with a, according to Art. 5.

2. Or find the value of $\log \sin B$. Then, if $\log \sin B < 10$, there is *one* solution when $a > b$, and *two* solutions if $a < b$. If $\log \sin B > 10$, the triangle is impossible.

EXERCISES XX.

1. In the triangle ABC, given $a = 75.5$, $b = 98.5$, $A = 37° 37'$; find B, C, and c.

SOLUTION.—Applying logarithms to the formulas given above, and substituting the numerical values, we have the following operations:

$a = 75.5$	$\log b = 1.993436$	$\log a = 1.877947$
$b = 98.5$	$\log \sin A = 9.785597$	$\log \sin C = 9.999989$
$A = 37° 37'$	$\text{colog } a = 8.122053$	$\text{colog } \sin A = 0.214403$
Here $a > b$ and	$\log \sin B = 9.901086$	$\log c = 2.092339$
$\log \sin B < 10$.	$B = 52° 46' 48''$	$c = 123.69$
$\therefore$ *one* solution.	$C = 89° 36' 12''$	

Note.—By constructing the triangle and examining it geometrically, it will be seen that there is but one solution.

2. In the triangle ABC, given $a = 150$, $b = 200$, $A = 44° 26'$; find B, C, and c.

Solution.—Substituting in the formulas given above, we have the following operations:

$a = 150$	$\log b = 2.301030$	$\log a = 2.176091$	2.176991
$b = 200$	$\log \sin A = 9.845147$	$\log \sin C = 9.962692$	9.618456
$A = 44° 26'$	$\text{colog } a = 7.823909$	$\text{colog } \sin A = 0.154853$	0.154853
Here $a < b$	$\log \sin B = 9.970086$	$\log c = 2.293636$	1.949400
$\log \sin B < 10$.	$B = 68° 58' 38''$	$c = 196.623$	
$\therefore$ *two* solutions.	or $111° 01' 22''$	or $c = 89.002$	
	$C = 66° 35' 22''$		
	or $24° 32' 38''$		

Note.—By constructing the triangle and examining it geometrically, it will be seen that there are two solutions. In Fig. 29, $AB'C = 68° 58' 38''$, and $ABC = 111° 01' 38''$; $ACB' = 66° 35' 22''$, $ACB = 24° 32' 38''$; $AB = 89.002$, $AB' = 196.623$.

3. In $\triangle ABC$, given $a = 62.50$, $c = 45.96$, $A = 79° 21'$; find $B = 54° 22' 22''$, $C = 46° 16' 38''$, $b = 51.69$.

4. In the triangle ABC, given $a = 15.71$, $b = 21.12$, $A = 27° 50'$; find the other parts.

Ans. $B = 38° 52' 47''$; $C = 113° 17' 13''$; $c = 30.906$;
or $B = 141° 7' 13''$; $C = 11° 2' 47''$; $c = 6.447$.

5. In the triangle ABC, given $a = 94.26$, $b = 126.72$, and $A = 27° 50'$; find the values of c, B, and C.

6. Given $a = 40$, $b = 80$, and $A = 30°$; find the other parts of the triangle.

7. Find the other parts of a triangle, given $a = 80$, $b = 100$, and $A = 60°$.

CASE III.

107. *Given two sides, a and b, and the included angle C, to find the remaining parts A, B, and c.*

METHOD.—In this case we proceed as follows:

1. To find A and B, we subtract C from 180° and divide by 2, which gives us the value of $\frac{1}{2}(A + B)$.

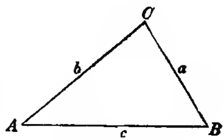

Fig. 30.

We then find $\frac{1}{2}(A - B)$ from Th. IV., which gives

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \times \tan \tfrac{1}{2}(A + B).$$

Then, $\quad \frac{1}{2}(A + B)$ plus $\frac{1}{2}(A - B) = A$.

And $\quad \frac{1}{2}(A + B)$ minus $\frac{1}{2}(A - B) = B$.

2. To find c, we apply Th. II., which gives

$$c = \frac{a \sin C}{\sin A}, \quad \text{or} \quad c = \frac{b \sin C}{\sin B}.$$

EXERCISES XXI.

1. In the triangle ABC, given $a = 680$, $b = 460$, and $C = 84°$; find the other parts of the triangle.

SOLUTION.—Following the method stated above, we have the following work:

$a + b = 1140$	$\log (a - b) = 2.342423$	$\log a = 2.832509$
$a - b = 220$	$\text{colog} (a + b) = 6.943095$	$\log \sin C = 9.997614$
$A + B = 96°$	$\log \tan \frac{1}{2} (A+B) = 10.045563$	$\text{colog} \sin A = 9.062046$
$\frac{1}{2} (A + B) = 48°$	$\log \tan \frac{1}{2} (A-B) = 9.331081$	$\log c = 2.892169$
$\frac{1}{2} (A-B) = 12° 5' 49''$	$\frac{1}{2} (A - B) = 12° 5' 49''$	$c = 780.134$
$A = 60° 5' 49''$		
$B = 35° 54' 11''$		

2. In the triangle ABC, given $a = 240$, $b = 360$, $C = 68° 36'$; find $A = 39° 21' 34''$, $B = 72° 02' 26''$, $c = 352.349$.

3. In the triangle ABC, given $a = 320$, $b = 562$, $C = 128° 04'$; find $A = 18° 21' 21''$, $B = 33° 34' 39''$, $c = 800$.

4. In the triangle ABC, given $b = 50.24$, $c = 43.25$, $A = 40° 15'$; find $B = 81° 24' 25''$, $C = 58° 20' 35''$, $a = 32.829$.

5. If two sides of a triangle are each equal to 60 ft., and the included angle is 60°, what is the third side?

6. If two sides of a triangle are each equal to 120 ft., and the included angle equals 120°, what is the third side?

CASE IV.

108. *Given the three sides, a, b, and c, of a plane triangle, to find the angles A, B, and C.*

METHOD.—Let fall a perpendicular upon the greater side from the angle opposite, dividing the triangle into two right triangles. Find the difference of the segments of the base by Theorem V.; half this difference added to half the base gives the greater segment, and subtracted from half the base gives the less.

We shall then have two sides and the right angle of two right triangles, from which we can find the acute angles by Theorem I.

EXERCISES XXII.

1. In a triangle ABC, given $AB = 60$, $AC = 50$, and $BC = 40$, to find the angles.

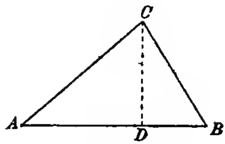
Fig. 31.

SOLUTION.—Let ABC denote the triangle; then $AB = 60$, $AC = 50$, $BC = 40$. Then, by Th. V.,

$$AB : AC + BC = AC - BC : AD - BD,$$
$$\text{or, } 60 : \quad 90 \quad = \quad 10 \quad : AD - BD,$$

Hence, $\quad AD - BD = 90 \times 10 \div 60 = 15;$

Then, $\quad AD = \frac{1}{2}(60 + 15) = 37.5,$

And $\quad BD = \frac{1}{2}(60 - 15) = 22.5.$

Then, in the triangle ACD, to find the angle BCD,

colog AC (50) $= 8.301030$	colog BC (40) $= 8.397940$
log AD (37.5) $= 1.574031$	log BD (22.5) $= 1.352183$
log sin $ACD = 9.875061$	log sin $BCD = 9.750123$
$ACD = 48° \ 35' \ 25''$	$\therefore BCD = 34° \ 13' \ 44''$

Hence, $\quad A = 90° - 48° \ 35' \ 25'' = 41° \ 24' \ 35'',$

$$B = 90° - 34° \ 13' \ 44'' = 55° \ 46' \ 16'',$$

$$C = 48° \ 35' \ 25'' + 34° \ 13' \ 44'' = 82° \ 49' \ 09''.$$

2. In a triangle ABC, given $a = 1005$, $b = 1210$, $c = 1368$; find the angles. *Ans.* 45° 22′ 34″; 58° 58′ 19″; 75° 39′ 7″.

3. In a triangle ABC, given $a = 340$, $b = 280$, and $c = 460$; find the angles.

Ans. $A = 47° \ 23' \ 16''$; $B = 37° \ 18' \ 31''$; $C = 95° \ 18' \ 13''$.

Another Method.

109. The angles of a plane triangle may also be found by means of the formulas given in Art. 76.

EXERCISES XXIII.

1. In the triangle ABC, the side $c = 800$, the side $b = 600$, and the side $a = 400$; required the three angles.

SOLUTION.—By Art. 76 we have

$$\sin^2 \tfrac{1}{2} A = \frac{(s-b)(s-c)}{bc}.$$

$s = \frac{1}{2}(800 + 600 + 400) = 900.$
$s - b = 900 - 600 = 300;$
$s - c = 900 - 800 = 100.$

We then find $\log(s-b)$, $\log(s-c)$, $\operatorname{colog} b$, and $\operatorname{colog} c$; their sum will be $\log \sin^2 \frac{1}{2} A$. Dividing by 2, we have $\log \sin \frac{1}{2} A = 9.397940$; from the Table, we find $\frac{1}{2} A = 14° \ 28' \ 39''$, whence, $A = 28° \ 57' \ 18''$.

OPERATION.

$\log(s-b)$ (300)	=	2.477121
$\log(s-c)$ (100)	=	2.000000
$\operatorname{colog} b$ (600)	=	7.221849
$\operatorname{colog} c$ (800)	=	7.096910
$\log \sin^2 \frac{1}{2} A$		2)18.795880
$\log \sin \frac{1}{2} A$	=	9.397940

$\frac{1}{2} A = 14° \ 28' \ 39''$

$A = 28° \ 57' \ 18''$

The other angles may be obtained in a similar manner from the formulas for $\sin^2 \frac{1}{2} B$ and $\sin^2 \frac{1}{2} C$.

The other two formulas of Art. 76, which may be used in this case, are

$$\cos^2 \tfrac{1}{2} A = \frac{s(s-a)}{bc}; \qquad \tan^2 \tfrac{1}{2} A = \frac{(s-b)(s-c)}{s(s-a)}.$$

NOTE.—Either of these three formulas may be used; but $\sin^2 \frac{1}{2} A$ is less accurate when the half angle is near 90°; and $\cos^2 \frac{1}{2} A$, when the half angle is near 0°; while $\tan^2 \frac{1}{2} A$ is applicable for any angle.

2. The three sides of a plane triangle are 20, 30, and 40; required the three angles.

Ans. 28° 57′ 18″; 46° 34′ 03″; 104° 28′ 39″.

3. In the triangle ABC, $a = 200$, $b = 250$, and $c = 300$; required the three angles.

Ans. 41° 24′ 35″; 55° 46′ 16″; 82° 49′ 09″.

Find the angles—

4. Given $a = 10$, $b = 24$, $c = 26$.
5. Given $a = 12$, $b = 12$, $c = 12$.
6. Given $a = 7$, $b = 8$, $c = 16$.
7. Given $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3} + 1$.

Solve the following without the use of logarithms:

8. If $b = 3$, $c = 2\sqrt{3}$, and $A = 30°$; prove $C = 90°$.
9. If $a = 2\sqrt{3}$, $b = 3 - \sqrt{3}$, and $c = 3\sqrt{2}$; prove $C = 120°$.
10. If $a = 2$, $b = 1 + \sqrt{3}$, and $c = \sqrt{6}$; prove $C = 60°$.
11. If $a = 12$, $b = \frac{399}{40}$, and $A = 45°$; prove $B = 36°$.
12. Find the angles of a triangle whose sides are in the ratio of 1, 2, and 3.

Remark.—All three angles may be computed by the formulas, and the accuracy of the results tested by seeing whether their sum equals 180°. For this method the formulas for the tangent may be put in a more convenient form. Thus, $\tan^2 \frac{1}{2} A$ may be written:

$$\frac{(s-a)(s-b)(s-c)}{s(s-a)^2} = \frac{1}{(s-a)^2}\left(\frac{(s-a)(s-b)(s-c)}{s}\right)$$

If we put

$$\frac{(s-a)(s-b)(s-c)}{s} = r^2, \text{ we have } \tan \tfrac{1}{2} A = \frac{r}{s-a}.$$

Similarly, $\tan \frac{1}{2} B = \frac{r}{s-b}$, and $\tan \frac{1}{2} C = \frac{r}{s-c}$.

In applying these formulas we may find the value of $\log r$, and use it in each one of the formulas in the computation, and thus slightly abridge the labor of computation.

SECTION VIII.

PRACTICAL APPLICATIONS.

HEIGHTS AND DISTANCES.

110. A **Horizontal Plane** is a plane which is parallel to the plane of the horizon.

111. A **Vertical Plane** is a plane which is perpendicular to a horizontal plane.

112. A **Horizontal Line** is any line in a horizontal plane. A *vertical line* is a line perpendicular to a horizontal plane.

113. A **Horizontal Angle** is an angle in a horizontal plane. A **Vertical Angle** is an angle in a vertical plane.

114. An **Angle of Elevation** is a vertical angle having one side horizontal, and the inclined side above the horizontal side; as *BAD*.

115. An **Angle of Depression** is a vertical angle having one side horizontal, and the inclined side under the horizontal side; as *CDA*.

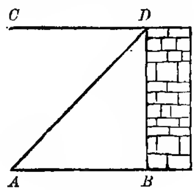

Fig. 32.

116. *Distances upon the ground* are usually measured by a chain, called *Gunter's Chain.* This chain is 4 rods or 66 feet long, and consists of 100 links. Sometimes a half chain is used, consisting of 50 links.

117. Angles are measured by various instruments. Horizontal angles are measured by an instrument called

The Compass. Horizontal and vertical angles are both measured by the *Theodolite*, or, what is still better for general use, a *Transit-Theodolite.*

Case I.

118. *To determine the height of a vertical object standing upon a horizontal plane.*

Method.—Measure from the foot of the object any convenient horizontal distance AB; at the point A take the angle of elevation BAC; then, in the triangle ABC we have a side and an acute angle; hence, we can readily find the altitude.

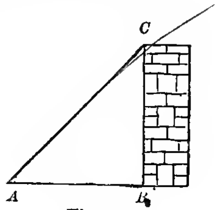

Fig. 33.

1. From the foot of a tower I measure a horizontal line 120 feet, and at its extremity find the angle of elevation to be 48° 36′; what was the height of the tower? *Ans.* 136.113 feet.

Case II.

119. *To find the distance of a vertical object whose height is known.*

Method.—Measure the angle of elevation to the top of the object, as before; we will then have a right triangle in which we know the perpendicular and an acute angle; hence, we can readily find the base.

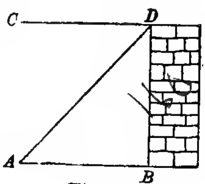

Fig. 34.

1. I took the angle of elevation to the top of a flagstaff whose height I knew to be 160 feet, and found it be 20°; how far was I from the staff?

Ans. 439.60 feet.

CASE III.

120. *To find the distance of an inaccessible object.*

METHOD.—Measure a horizontal base-line AB, and then take the angles formed by this line and lines from the object to the extremities of this base-line, as CAB and ABC; the distance AC or BC can then be readily found.

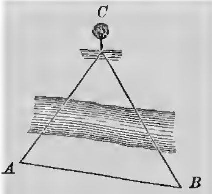

Fig. 35.

1. I am on one side of a river, and wish to know the distance to a tree on the other side. I measure 300 yards by the side of the river, and find that the two angles formed by this line and the lines from its extremities to the tree are 72° 40′ and 45° 36′ respectively; required the distance from each extremity of the base-line to the tree.

Ans. 243.362 yards; 325.15 yards.

CASE IV.

121. *To find the distance between two objects separated by an impassable barrier.*

METHOD.—Select any convenient station, as C, and measure the distance from it to each of the objects A and B and the angle C included between these lines. We can then readily find the distance AB.

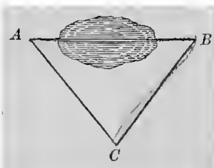

Fig. 36.

1. The distance between two trees cannot be directly measured: I therefore take a third position, from which each of the trees can be seen, and find the distances from it to the trees to be 300 and 250 yards re-

spectively, and the included angle 43° 16′; required the distance between the trees. *Ans.* 208.025 yards.

CASE V.

122. *To find the height of a vertical object standing upon an inclined plane.*

METHOD.—Measure any convenient distance DC on a line from the foot of the object, and at the point D measure the angles of elevation, EDA and EDB, to foot and top of the tower. By means of the two triangles DEA and DEB we can find the height of AB.

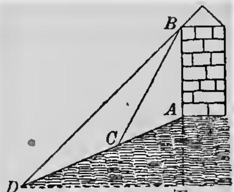

Fig. 37.

1. Wishing to determine the height of a tower situated upon a hill, I measured a distance down the slope of the hill 400 feet, and found the angles of elevation to the foot of the tower 42° 28′, and to the top of the tower 68° 42′; required the height of the tower. *Ans.* 486.747.

CASE VI.

123. *To find the height of an inaccessible object above a horizontal plane.*

FIRST METHOD.—Measure any convenient horizontal line AB directly toward the object, and take the angles of elevation at A and B; we will then have conditions sufficient to find DC.

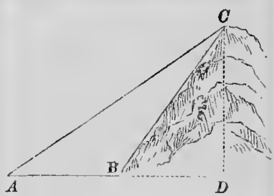

Fig. 38.

1. Wishing to find the altitude of a hill, I measured the angle of elevation at the bottom 60° 37′, and 460 feet from the foot, in a right line

from the top of the hill and the point at the foot, and in the same horizontal plane as the foot, I measured the angle of elevation 36° 52′; required the height of the hill.

Ans. 597.092.

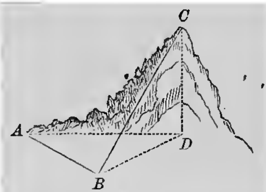

Fig. 39.

SECOND METHOD.—*If it is not convenient to measure a horizontal base-line toward the object*, we may measure any line AB, and also measure the horizontal angles BAD, ABD, and the angle of elevation DBC. Then, by means of the two triangles ABD and CBD, the height CD can be found.

CASE VII.

124. ***To find the distance between two inaccessible objects when points can be found from which both objects can be seen.***

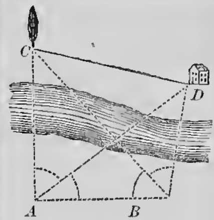

Fig. 40.

METHOD.—The method of measurement is indicated in the following problem. The method of solution we prefer leaving to the ingenuity of the pupil, that he may learn to think for himself.

1. Wishing to know the horizontal distance between a tree and house on the opposite side of a river, I took the following measurement:

$$AB = 400; \qquad CAD = 56° \ 30',$$
$$BAD = 42° \ 24'; \ ABC = 44° \ 36',$$
$$\text{and } DBC = 68° \ 50'.$$

Required the distance CD. *Ans.* 747.913.

Case VIII.

125. *To find the distance between two inaccessible objects when no points can be found from which both objects can be seen.*

Method.—The method is indicated in the following problem and figure. This case and the following one may be omitted with young students.

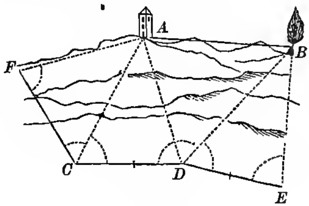

Fig. 41.

1. Wishing to know the horizontal distance between two inaccessible objects when no point can be found from which both objects can be seen, two objects C and D are taken, 600 feet apart, from the former of which A can be seen, from the latter B. From C we measure the distance CF, not in the direction DC, equal to 600 feet, and from D a distance DE equal to 600 feet. We then measure the following angles.

$$CFA = 80° \, 16', \; BED = 86° \, 25'.$$
$$ACF = 52° \, 24', \; BDE = 60° \, 24',$$
$$ACD = 56° \, 36', \; BDC = 150° \, 30'.$$

Required the distance AB. *Ans.* 1117.44 feet.

Case IX.

126. *To find the distances from a given point to three objects whose distances from each other are known.*

Method.—The method is indicated in the problem and figure.

1. I wish to locate three buoys, A, B, and C, in a harbor, so that the distance between A and B is 800 yards, between A and C 600 yards, between B and C 400 yards, and from a fixed point on shore the angle APC shall equal 33° 45′ and BPC 22° 30′; required the distances PA, PC, and PB.

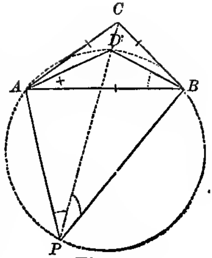

Fig. 42.

Ans. $PA = 710.193$; $PC = 1042.522$; $PB = 934.291$.

SECTION XI.

SUPPLEMENT.

127. The **Supplement** presents additional matter for those who wish to pursue the subject further.

Some Properties of Triangles.

The Right Triangle.

128. In the right triangle ABC, let b denote the base, a the altitude, c the hypotenuse, and M the area. Then, Geometry, B. IV., Th. 6,

$$M = \tfrac{1}{2}ab.$$

But, $a = b \tan A$, and $b = a \tan B$. Art. 26.

Hence $S = \tfrac{1}{2} b^2 \tan A$, and $M = \tfrac{1}{2} a^2 \tan B$. [43]

Hence, we can find the area from A and b or from a and B. From Ex. 1 and 2 below we can find the area having a and A or b and B. From Ex. 3 and 4 we can find the area, having given c and A or B.

EXERCISES XXIV.

Prove the following:

1. $M = \frac{1}{2} a^2 \cot A$.
2. $M = \frac{1}{2} b^2 \cot B$.
3. $M = \frac{1}{4} c^2 \sin 2A$.
4. $M = \frac{1}{4} c^2 \sin 2B$.

Find the other three parts:

5. Given B and c.
6. Given B and b.
7. Given B and a.
8. Given b and c.

The Isosceles Triangle.

129. In the isosceles triangle ABC, let h denote the altitude, a the equal sides, and c the base. Then we readily derive the relations given in the following exercises:

EXERCISES XXV.

In an isosceles triangle find the other parts—

1. Given a and A.
2. Given a and C.
3. Given c and A.
4. Given c and C.
5. Given h and A.
6. Given h and C.

Find the area—

7. Given a and A.
8. Given a and C.
9. Given a and c.
10. Given h and C.

The General Triangle.

130. In any triangle ABC, let c denote the base, a and b the two sides opposite the angles A and B respectively, and h the altitude.

Then, $\quad M = \frac{1}{2} ch; \quad$ but $\quad h = a \sin B.$

$$\left.\begin{aligned} &\text{Hence,} && M = \tfrac{1}{2} ac \sin B. \\ &\text{Similarly,} && M = \tfrac{1}{2} ab \sin C, \quad \text{and} \quad M = \tfrac{1}{2} bc \sin A. \end{aligned}\right\} \qquad [44]$$

Hence, *the area of a triangle is equal to one-half the product of any two of its sides into the sine of the included angle.*

131. A formula may also be found for the area when a side and two angles are given, the third angle being then known.

From Th. III., $\quad a = \dfrac{b \sin A}{\sin B}, \qquad c = \dfrac{b \sin C}{\sin B}.$

Substituting these values of a and c in $M = \frac{1}{2} ac \sin B$,

We have $$M = \frac{b^2 \sin A \sin C}{2 \sin B}.$$

Hence, *the area of a triangle is equal to the product of the sines of any two angles into the square of their included sides, divided by twice the sine of the third angle.*

132. A formula may also be derived for the area of a triangle when the three sides are given.

By For. [18], $\sin B = 2 \sin \frac{1}{2} B \cos \frac{1}{2} B.$

Substituting the values of $\sin \frac{1}{2} B$ and $\cos \frac{1}{2} B$, as given in Art. 76,

We have $$\sin B = \frac{2}{ac} \sqrt{s(s-a)(s-b)(s-s)}.$$

Substituting this value of $\sin B$ in [45],

We have $$M = \sqrt{s(s-a)(s-b)(s-c)}. \quad [45]$$

Hence, to find the area of a triangle when the three sides are given, *we subtract each side from the half sum of the sides, take the product of these differences and the half sum, and extract the square root of the product.*

EXERCISES XXVI.

Find the area of a triangle—

1. Given $a = 20$, $b = 30$, $C = 60°$.
2. Given $b = 30$, $c = 40$, $A = 45°$.
3. Given $a = 30$, $c = 40$, $B = 115°$.
4. Given $a = 40$, $b = 80$, $C = 48°$.
5. Given $a = 60$, $b = 80$, $c = 150°$.
6. Given $b = 100$, $A = 30°$, $B = 40°$.

The Radius of an Inscribed Circle.

133. Let ABC be any triangle whose sides are a, b, and c, and r the radius of the inscribed circle; then dividing the triangle into three triangles by drawing lines from the centre to the vertices of the three angles,

We have $M = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = \frac{1}{2}r(a+b+c)$.

Let $2s$ denote the sum of the three sides, and

We have $M = rs$; whence, $r = \frac{M}{s}$. [46]

Hence, *the radius of the inscribed circle is equal to the area of the triangle divided by one-half the sum of the sides.*

Cor.—Substituting in For. [46] the value of M given in [45], and reducing, we have

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}};$$

hence r as used on page 84 is equal to the radius of the inscribed circle.

Radius of a Circumscribed Circle.

134. Let ABC be circumscribed by a circle whose centre is O; and let R denote the radius. Draw $OD \perp$ to BC; then, $BD = DC$.

By Geometry, B. III., Th. 18, the angle $BOC = 2A$; hence, angle $BOD = A$, and $BD = R \sin BOD$, or $\frac{1}{2}a = R \sin A$.

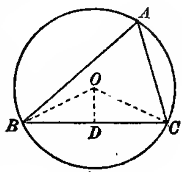

Fig. 43.

Hence, $a = 2R \sin A$.

From For. [44], $\sin A = \frac{2M}{bc}$.

Whence, $R = \frac{abc}{4M}$.

Therefore, *the radius of the circumscribed circle is equal to the product of the three sides of the triangle divided by four times its area.*

Cor.—From Art. 134 we have

$$2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Hence, *the diameter of the circumscribed circle is equal to the ratio of any side of a triangle to the sine of the opposite angle.*

EXERCISES XXVII.

Find the radius of an inscribed circle,

1. Given $a = 4$, $b = 5$, $c = 6$.
2. Given $a = 10$, $b = 20$, $A = 40°$.
3. Given $a = 30$, $b = 35$, $C = 30°$.
4. Given $a = 45$, $B = 45°$, $M = 24$.
5. Given $a = 30$, $C = 60°$, $M = 40$.
6. Given $a = b = c$.
7. Find the radius of a circumscribed circle in each of the above cases.
8. Find the angles of a right triangle if the hypotenuse is equal to four times one of the legs.
9. Find the legs of a right triangle if the hypotenuse is 12, and one acute triangle is twice the other.
10. Derive a formula for the area of a parallelogram, given two adjacent sides a and b and the included angle A.
11. Derive a formula for the area of an isosceles trapezoid, given the two parallel sides a and b and acute angle A.

Generalization of Angles.

135. We have Art. 54, $\sin A = 360° + A$, or $\sin A = 2 \times 180° + A$. If we add any number of times 360°, as n times 360°, the sine is still the same; hence $\sin A = 2n \times 180° + A$ or $\sin A = 2n\pi + A$.

136. Also $\sin A = \sin (180° - A)$ or $\sin (\pi - A)$. If we add any number of times 360°, as n times 360°, the sine is still the same; hence $\sin A = \sin (n\,360° + 180° - A) = \sin (2n \times 180° + 180° - A) = \sin \{(2n + 1)\,180° - A\} = \sin \{(2n + 1)\,\pi - A\}$. Therefore, if A_1 denotes the general value of an angle A whose sine is a, we have

$$A_1 = 2n\pi + A, \qquad \text{and} \qquad A_1 = (2n + 1)\pi - A.$$

137. From this we infer that *if two angles have the same sine, either their difference is an even multiple of π, or their sum is an odd multiple of π.*

138. Similarly, we may show that if A_1 denotes the general value of an angle A whose cosine is a, we have

$$A_1 = 2n\,180° \pm A, \quad \text{or} \quad A_1 = 2n\pi \pm A.$$

1. From this it is seen that, *if two angles have the same cosine, either their sum or their difference must be an even multiple of π.*

2. Similarly we may prove that, *if two angles have the same tangent, their difference must be some multiple of* π.

EXERCISES XXVIII.

What is the general value of an angle A

1. When $\sin A = \frac{1}{2}$?
2. When $\sin A = 1$?
3. When $\cos A = 1$?
4. When $\sin^2 A = \frac{1}{2}$?
5. When $\tan^2 A = \frac{1}{3}$?
6. When $\tan A = 1$?
7. When $\sec A = 2$?
8. When $\cot^3 A = -3\sqrt{3}$?
9. When $\csc^2 A = \frac{2}{3}$?
10. When $\tan^4 A = 9$?

11. Find the general values of A in the equation $\sin 3A = \sin A \cos 2A$.

SOLUTION.—We have $\sin(A + 2A) - \sin A \cos 2A = 0$; whence, $\cos A \sin 2A = 0$; hence, either $\cos A = 0$, or $\sin 2A = 0$. From the former we get $A =$ some odd multiple of $\frac{1}{2}\pi$, and from the latter we get $2A =$ any multiple of π. Hence, both are included in the equation $A = \frac{1}{2}n\pi$.

Find the general value of A in the following equations:

12. $\cos A = \cos 2A$.
13. $\sin 5A = 16 \sin^5 A$.
14. $\sin A + \cos A = \dfrac{1}{\sqrt{2}}$.
15. $\sin 9A - \sin A = \sin 4A$.
16. $\sin 4A + \sin 6A = 0$.
17. $\tan A + \cot A = 2$.
18. $\sec A = 2 \tan A$.
19. $\csc A \cot A = 2\sqrt{3}$.
20. $\sin A - \cos A = 4 \sin A \cos^2 A$.
21. $\tan(\frac{1}{4}\pi + A) = 1 + \sin 2A$.

Inverse Trigonometric Functions.

139. The expressions $\sin A = n$, $\cos A = n$, etc., may also be expressed thus: $A = \sin^{-1} n$ and $A = \cos^{-1} n$. To read these, notice that

$A = \sin^{-1} n$ is read, A equals the angle whose sine is n.

$A = \cos^{-1} n$ is read, A equals the angle whose cosine is n.

$A = \tan^{-1} n$ is read, A equals the angle whose tangent is n.

140. These are called *inverse trigonometric functions.* They are often found to be convenient in trigonometry.

NOTE.—The student will be careful to notice that in the expression $\sin^{-1}$, the (-1) is not to be regarded as an *exponent.*

141. Any relation which exists among trigonometrical functions may be expressed by means of the *inverse* notation.

EXERCISES XXIX.

1. What is the value of $\sin^{-1}\frac{1}{2}$?

SOLUTION.—Evidently $\sin^{-1}\frac{1}{2}$ equals 30°, since $\sin 30° = \frac{1}{2}$.

Find A, given

2. $A = \sin^{-1}\frac{1}{2}\sqrt{2}$.
3. $A = \cos^{-1}\frac{1}{2}\sqrt{3}$.
4. $A = \tan^{-1}\sqrt{3}$.
5. $A = \cos^{-1}\frac{1}{2}$.
6. $A = \cos^{-1}(-\frac{1}{2})$
7. $A = \cot^{-1} -\frac{1}{3}\sqrt{3}$.
8. $A = \sec^{-1}\sqrt{2}$.
9. $A = \csc^{-1}\frac{2}{3}\sqrt{3}$.

10. Given $A = \tan(\cos^{-1}\frac{2}{3})$, to find the value of A.

SOLUTION.—This means, what is the tangent of the arc whose cosine is $\frac{2}{3}$? Let x denote the angle; then $\cos x = \frac{2}{3}$, $\sin x = \sqrt{1-\frac{4}{9}} = \frac{1}{3}\sqrt{5}$; hence, $\tan x = \frac{1}{3}\sqrt{5} \div \frac{2}{3} = \frac{1}{2}\sqrt{5}$.

11. Given $\sin a = p$, and $\sin b = q$, to express $\sin(a+b)$ inversely.

SOLUTION.—Since $\sin a = p$, $\cos a = \sqrt{1-p^2}$; also $\cos b = \sqrt{1-q^2}$; hence $\sin(a+b) = p\sqrt{1-q^2} + q\sqrt{1-p^2}$.

Therefore, $$a + b = \sin^{-1}(p\sqrt{1-q^2} + q\sqrt{1-p^2}).$$

Or, $$\sin^{-1} p + \sin^{-1} q = \sin^{-1}(p\sqrt{1-q^2} + q\sqrt{1-p^2}).$$

Express similarly the inverse functions

12. Of $\sin(a-b)$.
13. Of $\cos(a+b)$.
14. Of $\cos(a-b)$.
15. Of $\tan(a+b)$.
16. Of $\tan(a-b)$.
17. Of $\cot(a+b)$.

Prove the following:

18. $\tan^{-1}\frac{3}{4} = 2\tan^{-1}\frac{1}{3}$.
19. $\sin^{-1}\frac{1}{5}\sqrt{5} + \tan^{-1}\frac{1}{3} = 45°$.
20. $2\tan^{-1}\theta = \tan^{-1}\dfrac{2\theta}{1-\theta^2}$.
21. $3\tan^{-1}\theta = \tan^{-1}\dfrac{3\theta-\theta^3}{1-3\theta^3}$
22. $\sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}) = 1$.
23. $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{63}{65}$.
24. $\sin^{-1}\theta = \cos^{-1}\sqrt{1-n^2}$.
25. $\sin^{-1}\theta + \cos^{-1}\theta = \dfrac{\pi}{2}$.
26. $\tan^{-1}\theta + \cot^{-1}\theta = \dfrac{\pi}{2}$.

27. $\sin\left(\tan^{-1}\frac{n}{m}\right) = \cos\left(\cot^{-1}\frac{n}{m}\right)\cdot$

28. $\sin^{-1}\frac{2\,ab}{a+b} = \tan^{-1}\frac{2\,ab}{a-b}\cdot$

29. $\tan^{-1}\frac{\sqrt{2}+1}{\sqrt{2}-1} - \tan^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}\cdot$

Solve the following equations:

30. $\sin^{-1}x + \sin^{-1}\frac{x}{2} = \frac{\pi}{4}\cdot$ 31. $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}\cdot$

32. $\sin^{-1}2x - \sin^{-1}x\sqrt{3} = \sin^{-1}x.$

33. $\sin 2x \cos^{-1}\cot 2\tan^{-1}x = 0.$

Extension of Functions of Two Angles.

142. In Art. 72 it was shown that the formulas for the sum of two angles hold when both angles are obtuse. We now show that they are true for all angles, positive or negative.

1. To do this we will show that they are true when one of the angles is increased by 90°. Thus, suppose $A^1 = 90° + A$; then by Art. 52,

$$\sin(A^1 + B) = \sin(90 + A + B) = \cos(A + B).$$

And, $\cos(A^1 + B) = \cos(90 + A + B) = -\sin(A + B).$

Hence, $\sin(A^1 + B) = \cos A\cos B - \sin A\sin B.$

$$\cos(A^1 + B) = -\sin A\cos B - \cos A\sin B.$$

Now, $\cos A = \sin(90° + A) = \sin A^1.$ Art. 52.

$$\sin A = -\cos(90° + A) = -\cos A^1.$$

Substituting, $\sin(A^1 + B) = \sin A^1\cos B + \cos A^1\sin B,$

$$\cos(A^1 + B) = \cos A^1\cos B - \sin A^1\sin B.$$

It is thus seen that Formulas [9] and [10] are true when one of the angles is increased by 90°; and in a similar way it may be shown that they are true for each increase of either or both angles by 90°, and therefore true for the sum of any two angles.

2. In a similar way it may be shown that the formulas for $\sin(A - B)$ and $\cos(A - B)$ are universal. Their universality may also be shown by deriving them from $\sin(A + B)$ and $\cos(A + B)$, which we have just shown are universal.

Thus, write $(A - B) + B = A$. Then by Art. 65,

$$\sin A = \sin[(A - B) + B] = \sin(A - B)\cos B + \cos(A - B)\sin B.$$
$$\cos A = \cos[(A - B) + B] = \cos(A - B)\cos B - \sin(A - B)\sin B.$$

Multiply the first equation by $\cos B$, and the second by $\sin B$,

$$\sin A \cos B = \sin(A - B)\cos^2 B + \cos(A - B)\sin B \cos B.$$
$$\cos A \sin B = -\sin(A - B)\sin^2 B + \cos(A - B)\sin B \cos B.$$

Subtracting, we have

$$\sin A \cos B - \cos A \sin B = \sin(A - B)(\sin^2 B + \cos^2 B).$$

But, $\sin^2 B + \cos^2 B = 1$; hence, transposing,

$$\sin(A - B) = \sin A \cos B - \cos A \sin B.$$

Hence the formula for $\sin(A - B)$, Art. 65, is universal.

3. In a similar manner, if we multiply the first equation by $\sin B$, and the second by $\cos B$, and add the results, and reduce, we shall obtain

$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

It is thus seen that Formulas [9], [10], [11], and [12] are general for all positive values of A and B; and therefore all the formulas derived from these are also general for all positive values of A and A.

4. Lastly, it may also be shown that these formulas are true for any negative values of A and B.

First, suppose C is negative, and less than A; then $A + B$ becomes $A - B$, and $A - B$ becomes $A + B$, and the formulas are true, as already shown. If A is negative, it merely changes the order of the letters.

Second, suppose B is negative and greater than A; then $A - B$ is negative. By Art. 44,

$$\sin(A - B) = -\sin(B - A) = -(\sin B \cos A - \cos B \sin A).$$

Whence, $\sin(A - B) = \sin A \cos B - \cos A \sin B.$

Also, $\cos(A - B) = \cos(B - A) = \cos B \cos A + \sin B \sin A.$

Whence, $\cos(A - B) = \cos A \cos B + \sin A \sin B.$

The same is also true if A is negative and greater than B.

Third, suppose A and B are both negative; and let $A = -A^1$ and $B = -B^1$.

$$\sin(A+B) = \sin(-A^1 - B^1) = -\sin(A^1 + B^1).$$
$$= -(\sin A^1 \cos B^1 + \cos A^1 \sin B^1).$$
$$= \sin(-A^1)\cos(-B^1) + \cos(-A^1)\sin(-B^1).$$
$$= \sin A \cos B + \cos A \sin B.$$

5. In the same manner Formulas [10], [11], and [12] may be shown to be true when both the angles are negative; hence all the formulas derived from these are also true. It is thus seen that the Formulas [9], [10], etc. are true for every value of A and B, positive or negative.

Application to the Circle.

143. We have regarded sine, cosine, tangent, etc. of an angle as a *ratio* of the sides of a right triangle, formed by the moving radius and its projection. This is the modern method of treating trigonometry; but the old method was to consider these functions as *lines* represented in a circle, and thus primarily as functions of arcs.

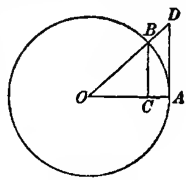

Fig. 44.

144. Thus, in the diagram, by the old system,

1. The line BC is the *sine* of the arc AB.
2. The line OC is the *cosine* of the arc AB.
3. The line AD is the *tangent* of the arc AB.
4. The line OD is the *secant* of the arc AB, etc.

145. In the old system the length of the sine, cosine, tangent, etc. depended upon the length of the radius of the circle; in the new system it is a fixed numerical value.

146. If the radius of the circle is taken as *unity*, the trigonometrical functions of angles according to the new system correspond with the trigonometrical functions of the arcs of a circle in the old system. Thus, denoting the angle AOC, or arc AB, by Z (see Fig. 44), we have, regarding $OB = 1$:

(1) $\sin Z = \frac{BC}{OB} = BC.$ (2) $\cos Z = \frac{OC}{OB} = OC.$

(3) $\tan Z = \frac{BC}{OC} = \frac{AD}{OA} = AD.$ (4) $\sec Z = \frac{OB}{OC} = \frac{OD}{OA} = OD.$

147. This graphic representation of the functions in the old system is for many minds simpler than the more abstract conception of ratios, and students should be familiar with it. The following definitions are given:

1. The **Sine** of an arc is the perpendicular drawn from its extremity to the diameter passing through its origin.

2. The **Cosine** of an arc is the distance between the foot of the sine of the arc and the centre of the circle.

3. The **Tangent** of an arc is the perpendicular to the radius at its origin, and limited by the radius produced passing through its extremity.

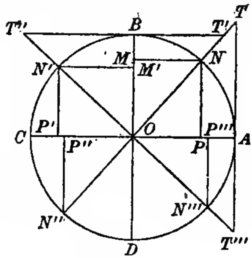

Fig 45.

4. The **Secant** of an arc is a line drawn from the centre of the circle through the extremity of the arc and limited by a tangent at its origin.

5. The **Cotangent** and **Cosecant** are respectively the tangent and secant of the complement of the arc.

148. The functions of arcs terminating in the different quadrants are represented in Fig. 45. Thus,

sin AON is NP; cos AON is OP;
tan AON is AT; sec AON is OT; etc.
sin AON' is $N'P'$; cos AON' is OP';
tan AON' is AT'''; sec AON' is OT'''; etc.
sin AON'' is $N''P''$; cos AON'' is OP'';
tan AON'' is AT; sec AON'' is OT; etc.

NOTE.—The student may be required to point out the lines of the circle which correspond to the formulas of Tables I., II., and III.

149. The relation of the values of the trigonometrical functions in the two systems is shown as follows. Let R denote the radius of the circle;

Then, $\sin angle\ BOC = \frac{BC}{R}$; and $BC = R \times \sin angle\ BOC$.

Also, $\sin arc\ AB = BC = R \times \sin angle\ BOC$.

Hence, $\sin angle\ BOC = \frac{\text{sin of the arc}}{\text{radius of circle}} = \frac{\sin O}{R}$.

150. Similar results hold for all the other trigonometrical functions of the two systems. Hence for any formula of the modern system which involves functions of *angles* we can readily deduce the corresponding formulas in the ancient system depending on the *arcs*, and *vice versâ*.

NOTES.—1. The modern method was introduced by Dr. Peacock, and has almost entirely superseded the ancient method.

2. The old definitions give some indications of the origin of the terms *sine*, *cosine*, etc. The word *sine* seems to have been derived from the Latin word *sinus*, a bosom. The arc is supposed to represent a bow, and thus gets its name; and the string, half of which represents the *sine* of half the arc, would come against the breast of the archer. The words *tangent* and *secant* are naturally derived from their definitions in Geometry.

EXERCISES XXX.

1. Construct the functions of an arc in quadrant II. Show their signs.

2. Construct the functions of an arc in quadrant III. Show their signs.

3. Construct the function of an arc in quadrant IV. Show their signs.

4. Required the signs of the functions of 250°; 320°; 400°; 450°: 600°; 800°.

5. Construct the angles less than 360° which have their sine equal to $\frac{5}{13}$; their cosine equal to $\frac{12}{13}$?

6. Construct the angles less than 270° which have $\sin A = \frac{4}{5}$; $\cos A = -\frac{2}{7}$; $\tan A = -\frac{3}{4}$; $\cot A = \frac{4}{7}$.

7. Limit the angle when sine and cosine are both positive or both negative. When cosine and tangent are both negative or both positive.

Miscellaneous Exercises.

Additional Formulas.

Note.—In 22-29, a, b, c denote the sides of a triangle opposite to the respective angles A, B, and C; and S denotes the half sum of the sides.

1. $\sin 40° + \sin 20° = \cos 10°.$
2. $\sin 80° - \sin 40° = \sin 20°.$
3. $1 + \sin A = \dfrac{\cos^2 A}{1 - \sin A}.$
4. $\sec A + \tan A = \dfrac{1}{\sec A - \tan A}.$
5. $\tan A + \cot A = 2 \csc 2A.$
6. $\tan A - \cot A = 2 \cot 2A.$
7. $\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A.$
8. $\sec^2 A + \csc^2 A = \sec^2 A \csc^2 A.$
9. $\sin 3A + \sin A = 2 \sin 2A \cos A.$
10. $\sin 3A - \sin A - 2 \cos 2A \sin A.$
11. $\cos 2A + \cos 4A = 2 \cos 3A \cos A.$
12. $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$
13. $\tan A + \tan B = \dfrac{\sin (A + B)}{\cos A \cos B}.$
14. $\tan A - \tan B = \dfrac{\sin (A - B)}{\cos A \cos B}.$
15. $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}.$
16. $\dfrac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A.$
17. $\dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin (A + B)}{\sin (A - B)}.$
18. $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B.$
19. $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \sec 2A - \tan 2A.$
20. $\dfrac{\cos A + \cos B}{\sin (A + B)} = \dfrac{\cos \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)}.$
21. $\dfrac{\cos B - \cos A}{\sin (A + B)} = \dfrac{\sin \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)}.$

22. $\dfrac{1-\tan A}{1+\tan A}=\sec 2A-\tan 2A.$

23. $\dfrac{\sin(A-B)}{\sin(A+B)}=\dfrac{(a+b)(a-b)}{c^2}.$

24. $\dfrac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}=\dfrac{a-b}{c}.$

25. $\dfrac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}=\dfrac{a+b}{c}.$

26. $\dfrac{\sin\frac{1}{2}A\sin\frac{1}{2}B}{\sin\frac{1}{2}C}=\dfrac{s-c}{c}.$

27. $\dfrac{\cos\frac{1}{2}A\cos\frac{1}{2}B}{\sin\frac{1}{2}C}=\dfrac{s}{c}.$

28. $\dfrac{\sin\frac{1}{2}A\cos\frac{1}{2}B}{\cos\frac{1}{2}C}=\dfrac{s-b}{c}.$

29. $\dfrac{\cos\frac{1}{2}A\sin\frac{1}{2}B}{\cos\frac{1}{2}C}=\dfrac{s-a}{c}.$

Functions of Special Angles.

Prove the following, remembering $\sin 15°=\sin(45°-30°)$.

1. $\sin 15°=\dfrac{\sqrt{3}-1}{2\sqrt{2}}.$

2. $\cos 15°=\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

3. $\tan 15°=2-\sqrt{3}.$

4. $\cot 15°=2+\sqrt{3}.$

5. Find $\sin 75°$; $\cos 75°$; $\tan 75°$; $\cot 75°$.

6. Find sine of $18°$.

Solution.—Let $A=18°$; then $2A=36°$, and $3A=54°$, and since $36°$ and $54°$ are complementary, we have $\sin 2A=\cos 3A$. Now, $\sin 2A=2\sin A\cos A$; and we can find $\cos 3A=4\cos^3 A-3\cos A$. Substituting and reducing, we find $\sin A=\frac{1}{4}(\sqrt{5}-1)$ $=\sin 18°$.

Prove the following:

7. $\cos 18°=\dfrac{\sqrt{(10+2\sqrt{5})}}{4}.$

8. $\cos 36°=\dfrac{1+\sqrt{5}}{4}.$

9. $\sin 36°=\dfrac{\sqrt{(10-2\sqrt{5})}}{4}.$

10. $\sin 9°=\dfrac{\sqrt{(3+\sqrt{5})}-\sqrt{(5-\sqrt{5})}}{4}.$

11. $\cos 9°=\dfrac{\sqrt{(3+\sqrt{5})}+\sqrt{(5-\sqrt{5})}}{4}$

12. $\tan 9° = \dfrac{6+2\sqrt{5}}{\sqrt{5}-1}$.

NOTE.—Similarly, since $18° - 15° = 3°$, we can find the functions of 3°, and from this of 6°, etc.

Find the value of x in the following equations:

13. $\sin 2x = \cos x$.

14. $\sin x + \cos x = \sqrt{2}$.

15. $\csc x = \csc \frac{1}{2}x$.

16. $\tan x + \tan(45° + x) = 2$.

17. $2\sin^2 x + \sin^2 2x = 2$.

18. $\tan(45° - x) + \cot(45° - x) = 4$.

19. $\tan(45° + x) = 3\tan(45° - x)$.

20. $\sin x \sin 3x = \frac{1}{2}$.

21. $\sin x + \cos x = \frac{1}{2}\sqrt{2}$.

22. $\sin x + \sin 4x = 0$.

23. $\cos 3x - \sin 3x = \frac{1}{2}\sqrt{2}$.

24. $\sqrt{3}\sin x - \cos x = \sqrt{2}$.

25. $6\cot^2 x - 4\cos^2 x = 1$.

26. $\sin 3x + \sin 2x + \sin x = 0$.

27. $\cos 3x + \cos 2x + \cos x = 0$.

28. $\sin^2 2x - \sin^2 x = \sin^2 30°$.

Additional Exercises.

1. Given $\sin x(\sin x - \cos x) = \frac{4}{25}$; find x.
2. Given $\tan x + \cot x = 2$; find x.
3. Given $\sin x + \cos 2x = \frac{1}{2}\sqrt{5}$, to find $\sin x$.
4. Given $4\sin x \sin 3x = 1$, to find $\sin x$.
5. Given $a\sin x + b\cos x = c$, to find $\sin x$.
6. Given $a\cos x = \tan x$, to find $\cos x$.

Change to forms for logarithmic computation:

7. $\tan x + \cot x$.
8. $\cot.\ x - \tan x$.
9. $\tan x + \cot y$.
10. $\cot x - \tan y$.
11. $1 + \tan x \tan y$.
12. $1 - \tan x \tan y$.
13. $\cot x \cot y + 1$.
14. $\cot x \cot y - 1$.

Demonstrate the following:

15. $\dfrac{1+\sin\theta}{1-\sin\theta} = \tan^2(45° - \frac{1}{2}\theta)$.

16. $\sin 4\theta = 4 \sin\theta \cos\theta - 8 \sin^3\theta \cos\theta$.

17. $\cos 4\theta = 1 - 8 \cos^2\theta + 8 \cos^4\theta$.

18. $\sin 3\theta = 4 \sin\theta \sin(60° - \theta) \sin(60° + \theta)$.

19. $\cos 3\theta = 4 \cos\theta \cos(60° - \theta) \cos(60° + \theta)$.

20. $\sin(a + b) \sin 3(a - b) = \sin^2(2a - b) - \sin^2(2b - a)$.

21. Find the value of $\cos(\sin^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2})$.

22. Find the value of $\tan(\tan^{-1} x + \cot^{-1} x)$.

23. Prove that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{8}$.

24. Prove that $\tan(2 \tan^{-1} a) = 2 \tan(\tan^{-1} a + \tan^{-1} a^3)$.

25. Prove $\tan^{-1}(\frac{1}{2} \tan 2x) + \tan^{-1}(\cot x) + \tan^{-1}(\cot^3 x) = 0$.

26. Find x, given $\tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$.

27. Prove $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$.

28. If $\sec a - \csc a = \frac{4}{3}$, prove that $a = \frac{1}{2} \sin^{-1} \frac{3}{4}$.

29. If $\sin A = \sin B$ *and* $\cos A = \cos B$, then either A and B are equal, or they differ by some multiple of four right angles.

30. If $\cos A = \cos B$ *and* $\sin A = -\sin B$, then $A + B$ is zero, or a multiple of four right angles, positive or negative.

31. The sum of the tangents of the three angles of a plane triangle is equal to their product.

32. In any plane triangle, $\cot \frac{1}{2} A + \cot \frac{1}{2} B + \cot \frac{1}{2} C = \cot \frac{1}{2} A \cot \frac{1}{2} B \cot \frac{1}{2} C$.

33. In a plane triangle, if $b = a \sin C$ and $c = a \cos B$, then the triangle is right-angled at A.

34. If the angles A, B, and C of a plane triangle are to each other as 2, 3, and 4, prove that $2 \cos \frac{1}{2} A = \frac{a + c}{b}$.

35. In any plane triangle ABC, if the angle made by a line drawn from the vertex C to the middle of the base c is denoted by Z, then

$$2 \cot Z = \cot A - \cot B.$$

SPHERICAL TRIGONOMETRY.

SECTION XII.

INTRODUCTORY DEFINITIONS.

151. Spherical Trigonometry treats of the solution of spherical triangles.

152. A **Spherical Triangle** is a portion of the surface of a sphere bounded by three arcs of great circles of the sphere.

153. The sides of a spherical triangle, being arcs of great circles, measure the plane angles formed by radii of the sphere drawn to the vertices of the triangle.

154. Each angle of a spherical triangle has the same measure as the dihedral angle included by the planes of its sides.

155. The sides of a spherical triangle may have any values from 0° to 360°; but in this treatise only sides less than 180° will be considered. The angles may have any values from 0° to 180°.

156. If two parts of a spherical triangle are both greater or both less than 90°, they are said to be in the *same quadrant;* but if one part is greater and the other less than 90°, they are said to be in *different quadrants.*

157. Spherical triangles are divided into two general classes—*right spherical triangles* and *oblique spherical triangles,*

the same as plane triangles. A right spherical triangle may have one, two, or even three right angles.

158. A spherical triangle having two right angles is called a *bi-rectangular triangle.* A spherical triangle having three right angles is called a *tri-rectangular triangle.* A spherical triangle having one or more sides equal to a quadrant is called a *quadrantal triangle.*

159. The nature of a spherical triangle will be seen by examining the diagram in the margin. The side AB measures the plane angle AOB; the side BC measures the plane angle BOC, and the side AC measures the plane angle AOC.

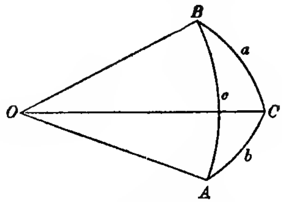

Fig. 46.

The spherical angle B is measured by the dihedral angle formed by the two planes AOB and COB; the spherical angle C is measured by the dihedral angle formed by the two planes AOC and BOC, etc.

160. It will be remembered, as shown in Geometry, that in every spherical triangle we have the following truths:

1. *The sum of the sides is less than* 360°.

2. *The sum of the angles is greater than* 180° *and less than* 540°.

3. *If two angles of a spherical triangle are equal, the opposite sides are equal.*

4. *If one angle of a spherical triangle is greater than another, the side opposite the greater angle is greater than the side opposite the less angle—and conversely.*

5. *The sides and angles of any spherical triangle are respectively the supplements of the angles and sides of the polar triangle.*

161. Thus, if the angles of a spherical triangle are denoted by A, B, C, and the sides opposite these angles respectively by a, b, c, and the corresponding angles and sides of the polar triangle by A', B', C', a', b', and c', then we shall have the following relations:

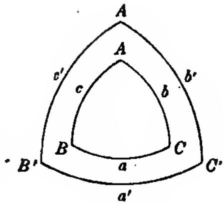

Fig. 47.

$$A = 180° - a'. \qquad C = 180° - c'. \qquad B' = 180° - b.$$
$$B = 180° - b'. \qquad A' = 180° - a. \qquad C' = 180° - c.$$

EXERCISES XXXI.

1. If the angles of a spherical triangle are 50°, 60°, and 85°, what are the sides of its polar triangle?

2. The sides of a spherical triangle are 75°, 97°, and 115°; what are the angles of the polar triangles?

3. What kind of a triangle is the polar triangle of a quadrantal triangle? *Ans.* A right triangle.

4. If a triangle has three right angles, what is the length of the sides of the triangle? *Ans.* Quadrants.

5. Prove that if a triangle has two right angles, the sides opposite these angles are quadrants, and the side opposite the third angle measures that angle.

6. Find the length of the sides of the polar triangle in Ex. 1, in units of length, if the diameter of the sphere is 8 units.

SECTION XIII.

THE RIGHT SPHERICAL TRIANGLE.

Fundamental Formulas.

162. We proceed first to find the relation of the functions of the sides and angles of a right spherical triangle.

Let ABC be a right spherical triangle, right angled at C; and let O be the centre of the sphere. Denote the angles by the letters A, B, and C, and their opposite sides by a, b, and c.

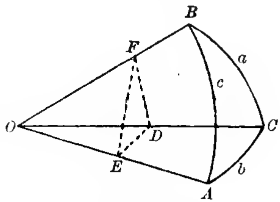

Fig. 48.

Draw OA, OB, and OC, each of which will be the radius of the sphere. From F draw $FE \perp$ to OA, and from E erect $ED \perp$ to OA, and draw FD; then the angle FED will measure the dihedral angle whose edge is OA; and angle $FED =$ angle A.

The plane FDE is $\perp$ to the plane AOC (B. VI. Th. 21); hence FD, the intersection of the planes FDE and FOC, is $\perp$ to the plane AOC (B. VI. Th. 24); therefore, FD is $\perp$ to OC and DE.

Now from the principles of Plane Trigonometry, and changing the form of some of the expressions by multiplying and dividing by the same quantity, we have the following results:

$$\frac{OE}{OF} = \frac{OE}{OD} \times \frac{OD}{OF}; \quad \text{that is,} \quad \cos c = \cos a \cos b. \quad [47]$$

$$\frac{FD}{OF} = \frac{FD}{FE} \times \frac{FE}{OF}; \text{ that is, } \mathbf{\sin a = \sin A \sin c.} \quad [48]$$

Similarly, $\mathbf{\sin b = \sin B \sin c.}$

Again,

$$\frac{DE}{EF} = \frac{DE}{OE} \times \frac{OE}{EF}; \text{ or } \mathbf{\cos A = \tan b \cot c.} \quad [49]$$

Similarly, $\mathbf{\cos B = \tan a \cot c.}$

Again,

$$\frac{ED}{OD} = \frac{ED}{FD} \times \frac{FD}{OD}; \text{ or } \mathbf{\sin b = \cot A \tan a.} \quad [50]$$

Similarly, $\mathbf{\sin a = \cot B \tan b.}$

Taking the product of the two formulas [50], we have

$$\frac{\sin a \sin b}{\tan a \tan b} = \cot A \cot B.$$

Whence [47], $\mathbf{\cos c = \cot A \cot B.}$ [51]

Multiply the first formula in [48] by the second in [49], we have

$$\sin a \cos B = \sin A \sin c \tan a \cot c.$$

Whence, $\cos B = \sin A \dfrac{\tan a}{\sin a} \times \sin c \cot c = \sin A \dfrac{\cos c}{\cos a}.$

Or, [47], $\mathbf{\cos B = \sin A \cos b.}$ [52]

Similarly, $\mathbf{\cos A = \sin B \cos a.}$

163. In deriving the formulas under Art. 162, it was assumed in the construction of the figure that all the parts of the triangle, except the right angle, are less than 90°. The formulas are, however, true for any right spherical triangle, as is readily seen.

Suppose one leg a to be greater than 90°. Then construct a figure (Fig. 49), as in Art. 162.

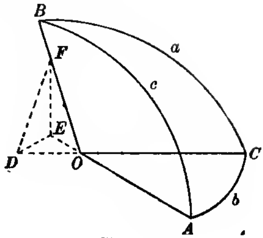

Fig. 49.

Now, $\frac{OE}{OF} = \frac{OE}{OD} \times \frac{OD}{OF}.$

Or,

$\cos(180° - c) = \cos b \cos(180° - a)$

Whence, $\cos c = \cos a \cos b$;

and this is the same as Formula [47].

Again, suppose that both legs a and b are greater than 90°. Construct the figure as before (see Fig. 50).

Then, $\frac{OE}{OF} = \frac{OE}{OD} \times \frac{OD}{OF}.$

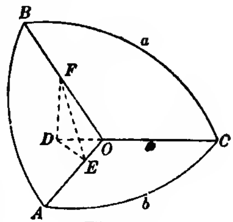

Fig 50.

Or,

$\cos c = \cos(180° - b) \cos(180° - a).$

Whence, $\cos c = \cos a \cos b.$

and this is the same as Formula [47].

Therefore the formulas of Art. 162 are universally true.

EXERCISES XXXII.

1. If $c = 90°$, what may be inferred in respect to the other parts?

Solution.—In For. [47] $\cos c = \cos a \cos b$. If $c = 90°$, $\cos c = 0$; hence, either $\cos a$ or $\cos b$ is 0, and either a or b is equal to 90°. If $a = 90°$, then $A = 90°$, and $B = b$. If $b = 90°$, then $B = 90°$, and $A = a$.

2. If $a = 90°$, what may be inferred in respect to the other parts? If $a = 90°$ and $c = 90°$? If $a = 90°$ and $b = 90°$?

3. What may be inferred in respect to the other parts if $b = 90°$? If $a = A$? If $b = B$, or $c = C$?

4. What will each of the formulas in Art. 162 become when applied to the polar triangle?

Formulas of Plane and Spherical Compared.

164. The six formulas of Art. 162, comprising ten equations, enable us to solve every case of right spherical triangles. Put in another form, as below, they may be remembered by their analogy to the corresponding formulas for plane triangles:

In Plane Right Triangle.	In Spherical Right Triangle.
$\sin A = \frac{a}{c}\cdot \quad \sin B = \frac{b}{c}\cdot$	$\sin A = \frac{\sin a}{\sin c}\cdot \quad \sin B = \frac{\sin b}{\sin c}\cdot$
$\cos A = \frac{b}{c}\cdot \quad \cos B = \frac{a}{c}\cdot$	$\cos A = \frac{\tan b}{\tan c}\cdot \quad \cos B = \frac{\tan a}{\tan c}\cdot$
$\tan A = \frac{a}{b}\cdot \quad \tan B = \frac{b}{a}\cdot$	$\tan A = \frac{\tan a}{\sin b}\cdot \quad \tan B = \frac{\tan b}{\sin a}\cdot$
$\sin A = \cos B. \quad \sin B = \cos A.$	$\sin A = \frac{\cos B}{\cos b}\cdot \quad \sin B = \frac{\cos A}{\cos a}\cdot$
$c^2 = a^2 + b^2.$	$\cos c = \cos a \cos b.$
$1 = \cot A \cot B.$	$\cos c = \cot A \cot B.$

Napier's Rules.

165. The ten formulas of Art. 162, by a very ingenious device, may all be embraced in two general rules, easily remembered and applied. These rules are due to Baron Napier, the distinguished inventor of logarithms.

166. In this device, five of the parts of the triangle are considered, the *two sides* about the right angle, the *complements* of their *opposite angles*, and the *complement* of the *hypotenuse*. These are called Napier's *Circular Parts*.

These parts are represented thus: **a, b, co. A, co. B,** and **co. C.** Notice that co. A equals 90° — A, etc.

167. Any one of these parts may be taken as the *middle*

part; and then the two parts adjacent to it are called *adjacent* parts, and those which are separated from it are called *opposite* parts.

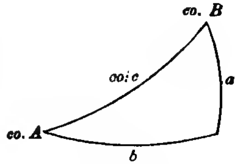

Fig. 51.

Thus, if co. C is taken as the *middle* part, then co. A and co. B are *adjacent* parts, and a and b are *opposite* parts, as is seen in the figure.

It will be noticed that the right angle does not enter as one of the parts, and that the two sides including it are regarded as adjacent.

168. The two rules of Napier are as follows:

RULE I. *The sine of the middle part is equal to the product of the* tangents *of the* adjacent *parts.*

RULE II. *The sine of the middle part is equal to the product of the* cosines *of the* opposite *parts.*

NOTE.—It will aid the memory to notice that the vowel o occurs in cosine and opposite, while a occurs in tangent and adjacent.

169. The correctness of these rules may be shown by taking each of the five parts as the middle part, and comparing the resulting equations with the formulas of Art. 162.

Thus, let co. c (see Fig. 51) be taken as the middle part; then co. A and co. B are the adjacent parts, and a and b are the opposite parts. Then by Napier's Rules we have

$$\sin(\text{co. } c) = \tan(\text{co. A}) \tan(\text{co. B}).$$

Whence, $\cos c = \cot \text{A} \cot \text{B}$.

Also, $\sin(\text{co. } c) = \cos a \cos b$.

Whence, $\cos c = \cos a \cos b$.

These results, it will be seen, correspond with Formulas

[51] and [47]; and in a similar manner all the formulas may be derived from the two rules.

NOTE.—The rules were originally derived from the formulas, and may be so derived by substituting for A, B, and C in the formulas, their complements.

EXERCISES XXXIII.

1. Derive Formulas [48] from Napier's Rules.

2. Derive Formulas [50] from Napier's Rules.

3. Derive Formulas [49] and [52] from Napier's Rules.

4. If we take for the five parts of the triangle the hypotenuse, the two oblique angles, and the complements of the legs, what formulas will Napier's Rules give?

5. On this supposition, what rules should we have to give the same results as Napier's Rules?

NOTE.—The rules thus derived are known as *Mauduit's Rules.*

The Ambiguous Cases.

170. In applying Napier's Rules, or the formulas of Art. 162, where the part sought is to be determined by the *sine*—the same sine corresponds to two different angles or arcs, supplements of each other—it becomes necessary to discover such a relation between the parts as will enable us to determine which of the two angles or arcs is to be taken.

171. For this purpose we shall prove the following principles:

PRIN. 1. *In a right spherical triangle, a side and its opposite angle are always in the same quadrant.*

For we have [52],

$$\sin A = \frac{\cos B}{\cos b}.$$

Now A is always less than 180°; hence sin A is always *plus*; therefore cos B and cos b must always have the same sign, and hence must both be greater or both less than 90°.

PRIN. 2. *If the two sides of a right spherical triangle, including the right angle, are in the same quadrant, the hypotenuse is less than* 90°; *but if the two sides are in different quadrants, the hypotenuse is greater than* 90°.

For we have [47],

$$\cos c = \cos a \cos b.$$

Now, if cos a and cos b have the same sign, cos c is *positive*, and hence c is less than 90°; but if cos a and cos b have different signs, cos c is *negative*, and hence c is greater than 90.°

NOTE.—These two principles enable us to determine the nature of the part to be found in every ease, exeept when an oblique angle and an opposite side are given to find the other parts. In that ease there may be two solutions, one solution, or no solution, as will be shown in the treatment of the ease.

Solution of Right Spherical Triangles.

172. In the solution of right spherical triangles there are six cases, as follows. Given,

1. The two legs.
2. The two angles.
3. The hypotenuse and one leg.
4. The hypotenuse and one angle.
5. One leg and its adjacent angle.
6. One leg and its opposite angle.

173. In solving these several cases the formulas given under Art. 162 may be taken from the book, or these formulas may be readily derived from Napier's Rules.

1. In applying Napier's Rules to obtain the formulas, it will be readily seen which of the three parts—the two given and the one

required—is to be taken as the middle part. Thus, if the three parts are all adjacent to one another, the middle one of the three is the *middle* part, and the other two are *adjacent* parts; if one is separated from the other two parts, then the part which stands by itself is the *middle* part, and the other two parts are *opposite* parts.

2. Thus, suppose we have a and b given to find the other parts, then to find c we write the terms, a, b, co. c, and since co. c is separated from a and b (see Fig. 52), we take co. c for the middle part, and have by Rule I.,

$$\sin (\text{co. } c) = \cos a \cos b,$$

or, $$\cos c = \cos a \cos b.$$

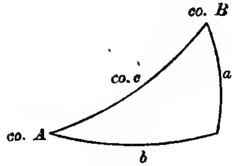

Fig. 52.

To find A we write a, b, and co. A, and since the terms are not separated (see Fig. 52), we take b for the middle part, and by Rule II. have

$$\sin b = \tan a \tan (\text{co. } A) = \tan a \cot A.$$

Whence, $\cot A = \cot a \sin b.$

Students are advised to derive the formulas from Napier's Rules.

Case I.

174. *Given the two legs a and b of the triangle ABC.*

1. In the spherical triangle ABC, right angled at C, $a = 59° 38'$ and $b = 48° 24'$; find A, B, and c.

Solution.—The formulas for the solution, derived by Napier's Rules or taken from Art. 162, are,

$$\cos c = \cos a \cos b. \quad [47]$$
$$\cot A = \cot a \sin b. \quad [50]$$
$$\cot B = \sin a \cot b. \quad [50]$$

Here C, A, and B are determined by the cosine, and there is no ambiguity.

Operation.

$$\log \cos a\ (59° 38') = 9.703749$$
$$\log \cos b\ (48° 24') = \underline{9.822120}$$
$$\log \cos c = 9.525869$$
$$c = 70° 23' 20''$$
$$\log \cot a\ (59° 38') = 9.767834$$
$$\log \sin b\ (48° 24') = \underline{9.873784}$$
$$\log \cot A = 9.641618$$
$$A = 66° 20' 23''$$

Similarly, $B = 52° 32' 48''$

EXERCISES XXXIV.

2. In the right spherical triangle, given $a = 75° 15'$ and $b = 120° 15'$; find $A = 77° 11' 14''$, $B = 119° 25' 17''$, $c = 97° 22' 9''$.

3. In the right spherical triangle ABC, given $a = 155° 27' 54''$, and $b = 29° 46' 8''$; find $c = 142° 9' 13''$, $A = 137° 24' 21''$, $B = 54° 1' 16''$.

CASE II.

175. ***Given the two oblique angles A and B of the triangle ABC.***

1. In the spherical triangle, right angled at C, $A = 62° 15'$, and $B = 56° 30'$; find a, b, and c.

SOLUTION.—The formulas for the solution, taken from Art. 162 or derived by Napier's Rules are,

$$\cos c = \cot A \cot B$$
$$\cos A = \cos a \sin B$$
$$\cos B = \cos b \sin A$$

From the second and third we have,

$$\cos a = \cos A \div \sin B$$
$$\cos b = \cos B \div \sin A,$$

which we use to find a and b.

OPERATION.

$$\begin{aligned}
\log \cot A \ (62° 15') &= 9.721089 \\
\log \cot B \ (56° 30') &= \underline{9.820783} \\
\log \cos c \quad &= 9.541872 \\
c &= 69° 37' 14'' \\
\log \cos A \ (62° 15') &= 9.668027 \\
\log \sin B \ (56° 30') &= \underline{9.921107} \\
\log \cos a \quad &= 9.746920 \\
a &= 56° \ 3' 25''
\end{aligned}$$

Similarly, $b = 51° 24' 56''$

EXERCISES XXXV.

2. In the right spherical triangle ABC, given $A = 69° 20'$, $B = 58° 16'$; find $a = 65° 28' 58''$, $b = 55° 47' 46''$, $c = 76° 30' 37''$.

3. In the right spherical triangle ABC, given $A = 47°$ $13' 43''$, $B = 126°$ $40'$ $24''$; find $a = 32°$ $08'$ $56''$, $b = 144°$ $27'$ $03''$, $c = 133°$ $32'$ $26''$.

Case III.

176. *Given the hypotenuse c and either leg a or b.*

1. In the spherical triangle ABC, right angled at C, $c = 56°$ $13'$, $a = 48°$ $30'$; find A, B, and b.

Solution.—The formulas for the solution taken from Art. 162, or derived by Napier's Rules, are,

$$\cos c = \cos a \cos b$$

$$\sin a = \sin A \sin c$$

$$\cos B = \tan a \cot c$$

Whence, $\cos b = \cos c \div \cos a$

and $\sin A = \sin a \div \sin c$

Operation.

$$\log \cos c \;(56°\ 13') = 9.745117$$
$$\log \cos a \;(48°\ 30') = \underline{9.821265}$$
$$\log \cos b = 9.923852$$
$$b = 32°\ 56'\ 49''$$

$$\log \sin a \;(48°\ 30') = 9.874456$$
$$\log \sin c \;(56°\ 13') = \underline{9.919677}$$
$$\log \sin A = 9.954779$$
$$A = 64°\ 18'\ 17''$$

Similarly, $B = 40°\ 52'\ 14''$

Note.—Two angles correspond to $\sin A$, but since a is less than $90°$, the angle A must also be less than $90°$ (Art. 170).

EXERCISES XXXVI.

2. In the right spherical triangle ABC, given $b = 37°$ $48'$ and $c = 66°$ $32'$; find $B = 41°$ $55'$ $34''$, $A = 70°$ $19'$ $18''$, and $a = 59°$ $44'$ $13''$.

3. In the right spherical triangle ABC, given $a = 95°$ $22'$ $30''$, $c = 91°$ $42''$; find $A = 95°$ $6'$, $B = 71°$ $36'$ $45''$, and $b = 71°$ $32'$ $12''$.

CASE IV.

177. *Given the hypotenuse c and either angle A or B.*

1. Given $c = 86° 50'$ and $A = 58° 30'$; find a, b, and B.

SOLUTION.—The formulas for the solution taken from Art. 162, or derived by Napier's Rules, are,

$$\sin a = \sin A \sin c$$
$$\tan b = \cos A \tan c$$
$$\cos c = \cot A \cot B$$

Whence, $\cot B = \cos c \tan A$

$$\log \sin A\ (58° 30') = 9.930766$$
$$\log \sin c\ (86° 50') = \underline{9.999336}$$
$$\log \sin a = 9.930102$$
$$a = 58° 21' 27''$$
$$\log \cos A\ (58° 30') = 9.718085$$
$$\log \tan c\ (86° 50') = \underline{11.257078}$$
$$\log \tan b = 10.975163$$
$$b = 83° 57' 21''$$

Similarly, $B = 84° 50' 56''$

EXERCISES XXXVII.

2. Given $c = 115° 35' 20''$, and $B = 110° 26' 30''$; find $a = 36° 6' 13''$, $b = 122° 18' 54''$, and $A = 40° 47' 35''$.

3. Given $c = 70° 23' 42''$ and $A = 66° 20' 40''$; find $a = 59° 38' 26''$, $b = 48° 24' 15''$, and $B = 52° 32' 55''$.

NOTE.—In Ex. 1, two values of a correspond to $\sin a$; but by Prin. I., a must be less than 90°, since A is less than 90°. Similarly, in Ex. 2, b must be greater than 90°.

CASE V.

178. *Given one leg a and its adjacent angle B.*

1. Given $a = 102° 30'$ and $B = 43° 24'$; to find b, c, and A.

SOLUTION.—The formulas for the solution derived by Napier's Rules or taken from Art. 162, are,

$$\tan b = \sin a \tan B$$
$$\cot c = \cot a \cos B$$
$$\cos A = \cos a \sin B$$

$$\log \sin a\ (102° 30') = 9.989582$$
$$\log \tan B\ (43° 24') = \underline{9.975732}$$
$$\log \tan b = 9.965314$$
$$b = 42° 42' 52''$$

Similarly, $c = 99° 9' 2''$

and $A = 98° 33' 9''$

EXERCISES XXXVIII.

2. Given $b = 42° \ 40' \ 24''$, and $A = 116° \ 36' \ 20''$; find $a = 126° \ 27' \ 47''$, $c = 115° \ 54' \ 35''$, and $B = 48° \ 54'$.

3. Given $a = 29° \ 46' \ 8''$, and $B = 137° \ 24' \ 21''$; find $A = 54° \ 1' \ 16''$, $b = 155° \ 27' \ 54''$, and $c = 142° \ 9' \ 13''$.

NOTE.—In Ex. 1, since cot a is negative, cot c is negative, and hence c is greater than 90°. In Ex. 2, a is greater than 90°, since A is greater than 90°.

CASE VI.

179. *Given one leg a and its opposite angle A.*

1. Given $a = 110° \ 32' \ 25''$ and $A = 98° \ 48' \ 50''$; find b, c, and B.

SOLUTION.—The formulas for the solution are

$$\sin b = \tan a \cot A$$

$$\sin c = \sin a \div \sin A$$

$$\sin B = \cos A \div \cos a$$

$$\begin{aligned} \log \tan a \, (110° \, 32' \, 25'') &= 10.426332 \\ \log \cot A \, (98° \, 48' \, 50'') &= \underline{\ 9.190490} \\ \log \sin b \qquad &= \ \ 9.616822 \end{aligned}$$

$\therefore b = 24° \, 26' \, 44''$ or $155° \, 33' \, 16''$

Similarly,

$c = 71° \, 22' \, 23''$ or $108° \, 37' \, 37''$

And, $B = 25° \, 53' \, 38''$ or $154° \, 6' \, 22''$

NOTE.—In this case, since all the required parts are determined by their sines, there are always two solutions. Thus, if in the triangle ABC, AB and AC are produced to meet in A', ABA' and ACA' are semi-circumferences, and the angle $A = A'$. The two triangles, ABC and $A'BC$, both have the two given parts a and A; but b', c', and B' in the second triangle are respectively the supplements of b, c, and B in the first triangle.

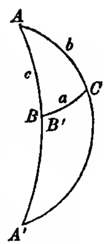

Fig. 53.

EXERCISES XXXIX.

2. Given, $A = 102°$ and $a = 120°$; find the other parts.

Ans. $\left.\begin{array}{l} b = 21° \ 36' \ 08'' \\ c = 62° \ 17' \ 51'' \\ B = 24° \ 34' \ 16'' \end{array}\right\}$ or $\left\{\begin{array}{l} b = 158° \ 23' \ 52'' \\ c = 117° \ 42' \ 09'' \\ B = 155° \ 25' \ 44'' \end{array}\right.$

3. Given $B = 80°$ and $b = 75°$; solve the triangle.

Ans. $\left.\begin{array}{l} a = 41° \ 09' \ 18'' \\ c = 78° \ 45' \ 45'' \\ A = 42° \ 08' \ 18'' \end{array}\right\}$ or $\left\{\begin{array}{l} a = 138° \ 50' \ 52'' \\ c = 101° \ 14' \ 15'' \\ A = 137° \ 51' \ 42'' \end{array}\right.$

Quadrantal Spherical Triangles.

180. A **Quadrantal Spherical Triangle** is one in which one side is equal to 90°. It is the polar triangle of some right spherical triangle.

181. To solve a quadrantal spherical triangle we pass to its polar triangle by subtracting each side and angle from 180°. The resulting polar triangle will be right angled, and may be solved as already explained. The parts of the given triangle may then be found by subtracting the parts of the polar triangle from 180.°

EXERCISES XL.

1. Given the quadrantal triangle ABC, in which $c = 90°$, $B = 42° \ 10'$ and $C = 115° \ 20'$.

SOLUTION.—Passing to the polar triangle $A'B'C'$, we have $C' = 90°$, $c' = 64° \ 40'$, and $b' = 137° \ 50'$.

Solving this triangle by the method for right triangles, we find $A' = 115° \ 23' \ 20''$, $B' = 132° \ 2' \ 13''$, and $a' = 125° \ 15' \ 36''$.

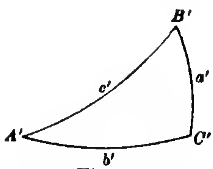

Fig. 54.

Subtracting each of these from 180°, we find the required parts of

the quadrantal triangle are $BC = 64° 36' 40''$, $AC = 47° 57' 47''$, and $A = 54° 44' 24''$.

2. Let ABC be a quadrantal triangle in which $c = 90°$, $A = 75° 42'$, and $b = 18° 37''$; find $C = 103° 34' 49''$, $B = 18° 04' 40''$, $a = 85° 28' 39''$.

3. In the spherical triangle ABC, given a, b, and c, each equal to $90°$, to find the angles.

182. An **Isosceles Triangle** is readily solved by dividing it into two right triangles by drawing an arc of a great circle from the vertex perpendicular to the base.

SECTION XIV.

THE OBLIQUE SPHERICAL TRIANGLE.

Fundamental Formulas.

183. We now proceed to find the relation of the functions of the sides and angles of an oblique spherical triangle.

I. *To find the relation of the sines of the sides and angles.*

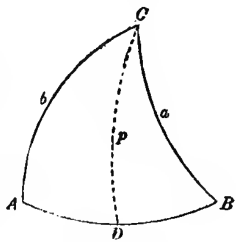

Fig. 55.

184. Let ABC, Fig. 54, be an oblique spherical triangle, A, B, C its three angles and a, b, c its three sides.

From C draw an arc CD of a great circle perpendicular to the side AB, meeting AB in D; and denote CD by p.

In the right triangles BCD and ACD, we have (Art. 162),

$$\sin p = \sin a \sin B$$

$$\sin p = \sin b \sin A$$

$$\left.\begin{array}{ll} \text{Hence,} & \sin a \sin B = \sin b \sin A \\ \text{Similarly,} & \sin a \sin C = \sin c \sin A \\ \text{And} & \sin b \sin C = \sin c \sin B \end{array}\right\} \quad [53]$$

These equations may be written in the form of proportions; as,

$$\sin a : \sin b = \sin A : \sin B.$$

Hence, we have the following theorem:

1. *The sines of the sides of a spherical triangle are proportional to the sines of their opposite angles.*

185. If in Fig. 54, the perpendicular CD cuts the side AB produced, we must have in place of $\sin A$, $\sin B$, or $\sin C$, $\sin\,(180° - A)$, $\sin\,(180° - B)$ or $\sin\,(180° - C)$. But these sines are equal to $\sin A$, $\sin B$, and $\sin C$, respectively (Art. 52); hence the formulas [53] are true for all cases.

II. *To find an expression for the cosines of the sides.*

186. In the triangle ABC, CD being perpendicular to the base as before, let $AD = m$ and $BD = n$.

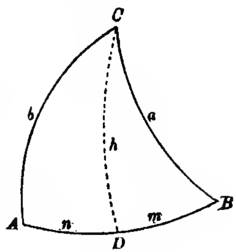

Fig. 56.

Now, in the right triangle BCD we have (Art. 162),

$$\cos a = \cos p \cos n = \cos p \cos\,(c - m)$$

Or,

$$\cos a = \cos p \cos c \cos m + \cos p \sin c \sin m.$$

But, Art. 162, $\cos p \cos m = \cos b.$

Whence, $\cos p = \cos b \sec m.$

And, $\cos p \sin m = \cos b \tan m.$

Or, Art. 162, $= \cos b \tan b \cos A.$

$= \sin b \cos A.$

Substituting these values of $\cos p \cos m$ and $\cos p \sin m$ in the second expression above, we obtain

$$\left.\begin{aligned} &\cos \mathbf{a} = \cos \mathbf{b} \cos \mathbf{c} + \sin \mathbf{b} \sin \mathbf{c} \cos \mathbf{A} \\ \text{Similarly, } &\cos \mathbf{b} = \cos \mathbf{a} \cos \mathbf{c} + \sin \mathbf{a} \sin \mathbf{c} \cos \mathbf{B} \\ \text{And, } &\cos \mathbf{c} = \cos \mathbf{a} \cos \mathbf{b} + \sin \mathbf{a} \sin \mathbf{b} \cos \mathbf{C} \end{aligned}\right\} [54]$$

These formulas give the following theorem:

2. *In any spherical triangle the cosine of each side is equal to the product of the cosines of the other two sides plus the product of the sines of these sides and the cosine of the included angle.*

III. *To find an expression for the cosines of the angles.*

187. Let $A'B'C'$ be the polar triangle of ABC, and denote its angles by A', B', and C', and its sides by a', b', c'. Then from Art. 186, we have

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

Now by Art. 161,

$$A' = 180° - a, \qquad B' = 180° - b, \qquad C' = 180° - C.$$
$$a' = 180° - A, \qquad b' = 180° - B, \qquad c' = 180° - C.$$

Substituting these values in the first formula [47], we have

$$-\cos A = (-\cos B)(-\cos C) - \sin B \sin C \cos a.$$

Whence by changing the signs, we have

$$\left.\begin{aligned} &\cos \mathbf{A} = \sin \mathbf{B} \sin \mathbf{C} \cos \mathbf{a} - \cos \mathbf{B} \cos \mathbf{C} \\ \text{Similarly, } &\cos \mathbf{B} = \sin \mathbf{C} \sin \mathbf{A} \cos \mathbf{b} - \cos \mathbf{C} \cos \mathbf{A} \\ \text{And, } &\cos \mathbf{C} = \sin \mathbf{A} \sin \mathbf{B} \cos \mathbf{c} - \cos \mathbf{A} \cos \mathbf{B} \end{aligned}\right\} [55]$$

188. In this way, by means of the polar triangle, any formula of a spherical triangle may be transformed into another in which angles take the place of sides and sides of angles.

189. By making one of the angles of the spherical triangle a right angle, all the formulas of a right spherical triangle, given in Art. 162, can be obtained from the formulas of an oblique spherical triangle.

EXERCISES XLI.

1. Show what formulas may be derived from [53] by making $A = 90°$; making $B = 90°$; making $C = 90°$.

2. Show what formulas may be derived from [53] by making $a = 90°$; making $A = B = 90°$; making $a = b = 90°$.

3. What formulas may be derived from [54] by making $A = 90°$; $B = 90°$; $C = 90°$; $a = 90°$; $b = 90°$; $c = 90°$?

IV. *To find expressions for half angles and sides.*

190. From the first equation of Art. 186, we deduce

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Subtracting this equation from unity, we have

$$1 - \cos A = \frac{\sin b \sin c + \cos b \cos c - \cos a}{\sin b \sin c}.$$

Substituting values as given in Arts. 69 and 70,

$$\sin^2 \tfrac{1}{2} A = \frac{\sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c}.$$

Let $s = \frac{1}{2}(a + b + c)$; then $\frac{1}{2}(a + b - c) = s - c$, and $\frac{1}{2}(a - b + c) = s - b$.

Substituting, we have

$$\left.\begin{aligned}
&\sin^2 \tfrac{1}{2} A = \frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}.\\
\text{Similarly, } &\sin^2 \tfrac{1}{2} B = \frac{\sin (s - c) \sin (s - a)}{\sin c \sin a}.\\
\text{And } \quad &\sin^2 \tfrac{1}{2} C = \frac{\sin (s - a) \sin (s - b)}{\sin a \sin b}.
\end{aligned}\right\} \quad [56]$$

191. If we add unity in Art. 190, and reduce as before, we may derive the following formulas:

$$\left.\begin{aligned}
\cos^2 \tfrac{1}{2} A &= \frac{\sin s (\sin s - a)}{\sin b \sin c}.\\
\cos^2 \tfrac{1}{2} B &= \frac{\sin s (\sin s - b)}{\sin c \sin a}.\\
\cos^2 \tfrac{1}{2} C &= \frac{\sin s \sin (s - c)}{\sin a \sin b}.
\end{aligned}\right\} \quad [57]$$

192. Dividing the corresponding formulas of Arts. 190 and 191, we have, For. [2],

$$\left.\begin{aligned}
\tan^2 \tfrac{1}{2} A &= \frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}.\\
\tan^2 \tfrac{1}{2} B &= \frac{\sin (s - c) \sin (s - a)}{\sin s \sin (s - b)}.\\
\tan^2 \tfrac{1}{2} C &= \frac{\sin (s - a) \sin (s - b)}{\sin s \sin (s - c)}.
\end{aligned}\right\} \quad [58]$$

193. Again, from the first equation of [55], we have

$$\cos a = \frac{\cos B \cos C + \cos A}{\sin B \sin C}.$$

Whence, $1 - \cos a = -\dfrac{\cos A + \cos (B + C)}{\sin B \sin C}.$

Hence, $\sin^2 \tfrac{1}{2} a = -\dfrac{\cos \frac{1}{2} (A + B + C) \cos \frac{1}{2} (B + C - A)}{\sin B \sin C}.$

Now let $s = \frac{1}{2}(A + B + C)$; then, $\frac{1}{2}(B + C - A) = (s - A)$;

$$\left.\begin{aligned} &\text{Whence,} \quad \sin^2 \tfrac{1}{2} a = \frac{-\cos S \cos (S - A)}{\sin B \sin C}. \\ &\text{Similarly,} \quad \sin^2 \tfrac{1}{2} b = \frac{-\cos S \cos (S - B)}{\sin C \sin A}. \\ &\text{And,} \quad \sin^2 \tfrac{1}{2} c = \frac{-\cos S \cos (S - C)}{\sin A \sin B}. \end{aligned}\right\} \quad [59]$$

194. In a similar manner we may find the following:

$$\left.\begin{aligned} \cos^2 \tfrac{1}{2} a &= \frac{\cos (S - B) \cos (S - C)}{\sin B \sin C}. \\ \cos^2 \tfrac{1}{2} b &= \frac{\cos (S - C) \cos (S - A)}{\sin C \sin A}. \\ \cos^2 \tfrac{1}{2} c &= \frac{\cos (S - A) \cos (S - B)}{\sin A \sin B}. \end{aligned}\right\} \quad [60]$$

195. And from [59] and [60] we derive, by For. [2],

$$\left.\begin{aligned} \tan^2 \tfrac{1}{2} a &= \frac{-\cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}. \\ \tan^2 \tfrac{1}{2} b &= \frac{-\cos S \cos (S - B)}{\cos (S - C) \cos (S - A)}. \\ \tan^2 \tfrac{1}{2} c &= \frac{-\cos S \cos (S - C)}{\cos (S - A) \cos (S - B)}. \end{aligned}\right\} \quad [61]$$

NOTES.—1. The second members of Formulas [59] and [61] must be essentially positive, though their algebraic sign is negative; for since $2\,S > 180°$, $S > 90°$ and $\cos S$ is negative; hence, $-\cos S$ is positive. Also, the positive sign must be given to the radical, since $\frac{2}{a}$ is less than a right angle.

2. Formulas [59], [60], and [61] might have been deduced by applying Formulas [56], [57], and [58] to the polar triangle.

Gauss's Equations.

196. From For. [9] we have

$$\sin \tfrac{1}{2}(A+B) = \sin \tfrac{1}{2}A \cos \tfrac{1}{2}B + \cos \tfrac{1}{2}A \sin \tfrac{1}{2}B.$$

Substituting the values of $\sin \frac{1}{2} A$, $\cos \frac{1}{2} B$, $\cos \frac{1}{2} A$, and $\sin \frac{1}{2} B$, derived from [56] and [57], and reducing by combining factors and extracting root, we have

$$\sin \tfrac{1}{2}(A+B) = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}} \times \sqrt{\frac{\sin s \sin(s-b)}{\sin c \sin a}}.$$

$$+ \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}} \times \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin c \sin a}}.$$

$$= \frac{\sin(s-a)+\sin(s-b)}{\sin c} \times \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin b}}.$$

Now, $\sin c = 2 \sin \frac{1}{2} c \cos \frac{1}{2} c$ [17], and $\sin(s-a) + \sin(s-b) = 2 \sin \frac{1}{2} c \cos \frac{1}{2}(b-a)$ [31]; and the quantity under the radical equals $\cos \frac{1}{2} C$ [57], hence,

$$\sin \tfrac{1}{2}(A+B) = \frac{2 \sin \frac{1}{2} c \cos \frac{1}{2}(b-a)}{2 \sin \frac{1}{2} c \cos \frac{1}{2} c} \times \cos \tfrac{1}{2} C.$$

Cancelling, multiplying by $\cos \frac{1}{2} c$, and reducing,

$$\sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2} c = \cos \tfrac{1}{2}(a-b) \cos \tfrac{1}{2} C.$$

Operating in the same way with the values of

$$\sin \tfrac{1}{2}(A-B),\ \cos \tfrac{1}{2}(A+B),\ \text{and}\ \cos \tfrac{1}{2}(A-B),$$

we have the four equations,

$$\left.\begin{aligned}
\sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2} c &= \cos \tfrac{1}{2}(a-b) \cos \tfrac{1}{2} C.\\
\cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2} c &= \cos \tfrac{1}{2}(a+b) \sin \tfrac{1}{2} C.\\
\sin \tfrac{1}{2}(A-B) \sin \tfrac{1}{2} c &= \sin \tfrac{1}{2}(a-b) \cos \tfrac{1}{2} C.\\
\cos \tfrac{1}{2}(A-B) \sin \tfrac{1}{2} c &= \sin \tfrac{1}{2}(a+b) \sin \tfrac{1}{2} C.
\end{aligned}\right\} \quad [62]$$

These four formulas are called *Gauss's Equations*, though, as Todhunter remarks, they are really due to *Delambre*.

Napier's Analogies.

197. By dividing the first of Gauss's Equations by the second, the third by the fourth, the fourth by the second, and the third by the first, we obtain the following equations:

$$\left.\begin{aligned}
\tan \tfrac{1}{2}(A+B) &= \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.\\
\tan \tfrac{1}{2}(A-B) &= \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.\\
\tan \tfrac{1}{2}(a+b) &= \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c.\\
\tan \tfrac{1}{2}(a-b) &= \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c.
\end{aligned}\right\} \quad [63]$$

198. Writing these equations in the form of proportions, we have,

$$\left.\begin{aligned}
\sin\tfrac{1}{2}(a+b) : \sin\tfrac{1}{2}(a-b) &= \cot\tfrac{1}{2}C : \tan\tfrac{1}{2}(A-B).\\
\cos\tfrac{1}{2}(a+b) : \cos\tfrac{1}{2}(a-b) &= \cot\tfrac{1}{2}C : \tan\tfrac{1}{2}(A+B).\\
\sin\tfrac{1}{2}(A+B) : \sin\tfrac{1}{2}(A-B) &= \tan\tfrac{1}{2}c : \tan\tfrac{1}{2}(a-b).\\
\cos\tfrac{1}{2}(A+B) : \cos\tfrac{1}{2}(A-B) &= \tan\tfrac{1}{2}c : \tan\tfrac{1}{2}(a+b).
\end{aligned}\right\} \quad [64]$$

These proportions are called, from their inventor, **Napier's Analogies.**

NOTE.—As is seen, there is a very intimate relation between Gauss's Equations and Napier's Analogies. We have derived the Analogies from the Equations; but the Analogies may be derived by an independent process, and the Equations deduced from the Analogies.

199. By examining the formulas [63] we reach the following conclusions:

1. In the first formula the factors $\cos \frac{1}{2}(a-b)$ and $\cot \frac{1}{2} C$ are always *positive;* hence $\tan \frac{1}{2}(A+B)$ and $\cos \frac{1}{2}(a+b)$ must always have the *same* sign. Therefore, if $a+b<180°$, and consequently $\cos \frac{1}{2}(a+b)>0$, then it follows that $\tan \frac{1}{2}(A+B)>0$, and therefore $A+B<180°$. Similarly, it follows that if $a+b>180°$, then also $A+B>180°$.

2. Also, if $a+b=180°$, and consequently $\cos \frac{1}{2}(a+b)=0$, then $\tan \frac{1}{2}(A+B)=\infty$; whence, $\frac{1}{2}(A+B)=90°$, and $A+B=180°$.

3. Conversely, it may be shown from the third formula that $a+b$ is less than, greater than, or equal to 180°, according as $A+B$ is less than, greater than, or equal to 180°.

Solution of Oblique Spherical Triangles.

200. In the solution of oblique spherical triangles, there are six cases, as follows. Given,

1. Two sides and their included angle.
2. Two angles and their included side.
3. Two sides and an angle opposite to one of them.
4. Two angles and a side opposite to one of them.
5. The three sides.
6. The three angles.

Case I.

201. *Given two sides, a and b, and the included angle C.*

Method.—We find the angles A and B by the first and second of Napier's Analogies, viz.:

$$\tan \tfrac{1}{2}(A+B)=\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C.$$

$$\tan \tfrac{1}{2}(A-B)=\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C.$$

The side c may then be found by [53], or by the third or fourth of Napier's Analogies. It is better, however, to find c from one of Gauss's Equations, since they involve functions of the same angles that are used in the two formulas of Napier's Analogies. We can use any one of the formulas; thus from the second we have

$$\cos \tfrac{1}{2} c = \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(A+B)} \sin \tfrac{1}{2} C.$$

EXERCISES XLII.

1. In a spherical triangle, given $a = 72° \ 36'$, $b = 40° \ 44'$, and $C = 54° \ 40'$; find the other parts.

SOLUTION.—

$$\begin{aligned} a &= 72° \ 36'. & \text{hence,} \quad \tfrac{1}{2}(a-b) &= 15° \ 56'. \\ b &= 40° \ 44'. & \tfrac{1}{2}(a+b) &= 56° \ 40'. \\ C &= 54° \ 40'. & \tfrac{1}{2} C &= 27° \ 20' \end{aligned}$$

$$\begin{aligned} \log \cos \tfrac{1}{2}(a-b) &= 9.982986 \\ \text{colog} \cos \tfrac{1}{2}(a+b) &= 0.260025 \\ \log \cot \tfrac{1}{2} C &= 10.286614 \\ \hline \log \tan \tfrac{1}{2}(A+B) &= 10.529625 \\ \tfrac{1}{2}(A+B) &= 73° \ 32' \ 39'' \\ \hline \log \cos \tfrac{1}{2}(a+b) &= 9.739975 \\ \text{colog} \cos \tfrac{1}{2}(A+B) &= 0.547790 \\ \log \sin \tfrac{1}{2} C &= 9.661970 \\ \hline \log \cos \tfrac{1}{2} c &= 9.949735 \\ \tfrac{1}{2} c &= 27° \ 02' \ 16'' \end{aligned}$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(a-b) &= 9.438572 \\ \text{colog} \sin \tfrac{1}{2}(a+b) &= 0.078060 \\ \log \cot \tfrac{1}{2} C &= 10.286614 \\ \hline \log \tan \tfrac{1}{2}(A-B) &= 9.803246 \\ \hline \tfrac{1}{2}(A-B) &= 32° \ 26' \ 37'' \\ \tfrac{1}{2}(A+B) &= 73° \ 32' \ 39'' \\ A &= 105° \ 59' \ 16'' \\ B &= 41° \ 06' \ 02'' \\ c &= 54° \ 04' \ 32'' \end{aligned}$$

2. Given $a = 80° \ 32' \ 40''$, $b = 120° \ 27' \ 18''$, $C = 48° \ 12' \ 21''$; find $A = 57° \ 9' \ 4''$, $B = 132° \ 45' \ 46''$, $C = 61° \ 5' \ 4''$.

3. Given $a = 124° \ 50' \ 48''$, $c = 75° \ 35' \ 50''$, $B = 56° \ 36' \ 26''$; find $A = 134° \ 10' \ 34''$, $C = 57° \ 49' \ 36''$, $b = 72° \ 49' \ 18''$.

CASE II.

202. *Given two angles, A and B, and the included side c.*

METHOD.—We find the sides a and b by the third and fourth of Napier's Analogies:

$$\tan \tfrac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \tfrac{1}{2} c.$$

$$\tan \tfrac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \tfrac{1}{2} c.$$

The angle C may then be found by the first or second of Napier's Analogies, or by one of Gauss's Equations. Thus, the first gives

$$\cos \tfrac{1}{2} C = \frac{\sin \frac{1}{2}(A+B)}{\cos \frac{1}{2}(a-b)} \cos \tfrac{1}{2} c.$$

EXERCISES XLIII.

1. In a spherical triangle, given $A = 108° 36' 45''$, $B = 40° 38' 28''$, $c = 56° 42' 22''$; find the other parts.

SOLUTION.—$A = 108° 36' 45''$.
$B = 40° 38' 28''$.
$c = 56° 42' 22''$.

$\frac{1}{2}(A-B) = 33° 59' 8\frac{1}{2}''$.
$\frac{1}{2}(A+B) = 74° 37' 36\frac{1}{2}''$.
$\frac{1}{2} c = 28° 21' 11''$.

$\log \cos \frac{1}{2}(A-B) = 9.918647$	$\log \sin \frac{1}{2}(A-B) = 9.747401$
$\operatorname{colog} \cos \frac{1}{2}(A+B) = 0.576582$	$\operatorname{colog} \sin \frac{1}{2}(A+B) = 0.015824$
$\log \tan \frac{1}{2} c = 9.732103$	$\log \tan \frac{1}{2} c = 9.732103$
$\log \tan \frac{1}{2}(a+b) = 10.227332$	$\log \tan \frac{1}{2}(a-b) = 9.495328$
$\frac{1}{2}(a+b) = 59° 21' 16''$	$\frac{1}{2}(a-b) = 17° 22' 19''$
$\log \sin \frac{1}{2}(A+B) = 9.984176$	$\frac{1}{2}(a+b) = 59° 21' 16''$
$\operatorname{colog} \cos \frac{1}{2}(a-b) = 0.020276$	$a = 76° 43' 35''$
$\log \cos \frac{1}{2} c = 9.944501$	$b = 41° 58' 57''$
$\log \cos \frac{1}{2} C = 9.948953$	$C = 54° 28' 40''$
$\frac{1}{2} C = 27° 14' 20''$	

2. Given $A = 130° 27' 38''$, $B = 110° 43' 20''$, $c = 124°$

$26' \, 37''$; find $a = 125° \, 55' \, 41''$, $b = 84° \, 30' \, 55''$, $C = 129° \, 12' \, 22''$.

3. Given $B = 148° \, 24' \, 36''$, $C = 86° \, 38' \, 42''$, $a = 88° \, 30' \, 47''$; find $b = 148° \, 21' \, 3''$, $c = 89° \, 30' \, 25''$, $A = 86° \, 21' \, 50''$.

CASE III.

203. *Given two sides a and b, and the angle A opposite one of them.*

METHOD.—The angle B is found from [53], from which we have

$$\sin B = \frac{\sin b \sin A}{\sin a}.$$

Then C and c may be found from the fourth and second of Napier's Analogies, which give

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} \tan \tfrac{1}{2}(a - b).$$

$$\cot \tfrac{1}{2} C = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} \tan \tfrac{1}{2}(A - B).$$

NOTE.—In this case, since B is found from the sine, there will sometimes be two solutions. If it is seen in the problem that $B < 90°$, there is but one solution. If in the calculation we find $\sin B > 1$, the problem is impossible.

The following truths may be readily deduced:

1st. When $A = 90°$, there is only *one solution*, and may be *no solution.*

2d. When $A < 90°$, there are *two solutions* when $a + b < 180°$, and $a < b$.

3d. When $A > 90°$, there are *two solutions* when $a + b > 180°$, and $a > b$.

EXERCISES XLIV.

1. Given $a = 53° 25'$, $b = 34° 26'$, and $A = 106° 35'$; find B, c, and C.

SOLUTION.—In this problem

$$
\begin{array}{lr}
\text{we have} & A > 90°, \\
\text{and} & a + b < 180°, \\
\text{hence,} & A + B < 180°; \\
\text{whence,} & B < 90°,
\end{array}
$$

and there is only one solution.

$$
\begin{aligned}
a + b &= 87° 51' \\
a - b &= 18° 59' \\
A + B &= 149° 01' 43'' \\
A - B &= 64° 08' 17'' \\
\log \sin \tfrac{1}{2} (A + B) &= 9.983941 \\
\log \tan \tfrac{1}{2} (a - b) &= 9.223218 \\
\text{colog} \sin \tfrac{1}{2} (A - B) &= \underline{0.274954} \\
\log \tan \tfrac{1}{2} c &= 9.482113 \\
\tfrac{1}{2} c &= 16° 52' 53'' \\
c &= 33° 45' 46''
\end{aligned}
$$

$$
\begin{aligned}
\log \sin A \ (106° 35') &= 9.981549 \\
\log \sin b \ (\ 34° 36') &= 9.752392 \\
\text{colog} \sin a \ (\ 53° 25') &= \underline{0.095289} \\
\log \sin B &= 9.829230 \\
B &= 42° 26' 43'' \\
\tfrac{1}{2} (a + b) &= 43° 55' 30'' \\
\tfrac{1}{2} (a - b) &= 9° 29' 30'' \\
\tfrac{1}{2} (A + B) &= 74° 30' 51\tfrac{1}{2}'' \\
\tfrac{1}{2} (A - B) &= 32° 04' 08\tfrac{1}{2}'' \\
\log \sin \tfrac{1}{2} (a + b) &= 9.841181 \\
\log \tan \tfrac{1}{2} (A - B) &= 9.796953 \\
\text{colog} \sin \tfrac{1}{2} (a - b) &= \underline{0.782768} \\
\log \cot \tfrac{1}{2} C &= 10.420902 \\
\tfrac{1}{2} C &= 20° 46' 36'' \\
C &= 41° 33' 12''
\end{aligned}
$$

2. Given $a = 75° 27' 40''$, $b = 118° 45' 36''$, $A = 84° 52' 34''$; find $B = 115° 34' 27''$, $c = 111° 45' 16''$, $C = 107° 07' 24''$.

3. Given $b = 40° 16'$, $c = 47° 44'$, $B = 52° 30'$; find $C = 65° 16' 35''$, $a = 53° 19' 20''$, $A = 79° 52' 22''$; or, $C = 114° 43' 25''$, $a = 14° 18' 22''$, $A = 17° 39' 22''$.

4. Given $a = 40° 20'$, $b = 60° 30'$, and $A = 50° 45'$; show that the solution is impossible.

Case IV.

204. *Given two angles, A and B, and the side a opposite one of them.*

Method.—The side a is found from [53], from which we have

$$\sin b = \frac{\sin a \sin B}{\sin A}.$$

Then c and C may be found from the fourth and second of Napier's Analogies, which give

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} \tan \tfrac{1}{2}(a-b).$$

$$\cot \tfrac{1}{2} C = \frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} \tan \tfrac{1}{2}(A-B).$$

Note.—In this case, since b is found from the sine, there will sometimes be two solutions, and may be no solution. If it is seen in the problem that $b < 90°$, there will be but one solution. If in the calculation we find $\sin b > 1$, there will be no solution.

The following truths may be readily deduced:

1st. When $a = 90°$, there is only *one solution*, and may be *no solution.*

2d. When $a < 90°$, there are *two solutions* when $A + B < 180°$, and $A < B$.

3d. When $a > 90°$, there are *two solutions* when $A + B > 180°$, and $A > B$.

EXERCISES XLV.

1. Given $A = 112° \ 50'$, $B = 135° \ 25'$, $a = 150° \ 36'$; find $b = 158° \ 2' \ 40''$, $c = 30° \ 45' \ 26''$, $C = 73° \ 46' \ 46''$.

2. Given $A = 114° \ 36' \ 40''$, $B = 82° \ 27' \ 18''$, $b = 86° \ 20' \ 30''$; find $a = 113° \ 45' \ 44''$, $c = 82° \ 7' \ 18''$, $C = 79° \ 44' \ 2''$.

3. Given $A = 132° 16'$, $B = 139° 44'$, $b = 127° 30'$; find $a = 65° 16' 35''$, $C = 165° 41' 38''$, $c = 162° 20' 38''$; or, $a = 114° 43' 25''$, $C = 126° 40' 40''$, $c = 100° 7' 38''$.

4. Given $A = 60° 30'$, $B = 40° 24'$, $b = 50° 36'$; show that the solution is impossible.

CASE V.

205. *Given the three sides, a, b, and c.*

METHOD.—The angles may be found by the Formulas [56] or [57] or [58]. The formulas for the tangent, however, are, generally, to be preferred.

The formulas for the tangent may be put in a still more convenient form by making

$$\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s} = \tan^2 r,$$

and substituting this in each and reducing, by which we obtain

$$\left.\begin{aligned}\tan\tfrac{1}{2}A &= \tan r \div \sin(s-a).\\ \tan\tfrac{1}{2}B &= \tan r \div \sin(s-b).\\ \tan\tfrac{1}{2}C &= \tan r \div \sin(s-c).\end{aligned}\right\} \quad [65]$$

NOTES.—1. When only one angle is to be found, use For's. [56], [57], or [58]; when all three angles are required, use For's. [65].

2. No ambiguity can arise in this case, since the half angles must be less than 90°.

EXERCISES XLVI.

1. Given $a = 60° 34' 20''$, $b = 48° 45' 26''$, $c = 76° 48' 53''$; find A, B, and C.

SOLUTION.—The solution by Formula [58] is as follows:

$a = 60° 34' 20''$	$\log \sin (s - b) = 9.844229$
$b = 48° 45' 26''$	$\log \sin (s - c) = 9.447084$
$c = \underline{76° 48' 53''}$	$\text{colog} \sin (s - a) = 0.269786$
$2s = 186° 08' 39''$	$\text{colog} \sin s = \underline{0.000625}$
$s = 93° 04' 19\frac{1}{2}''$	$2)19.561724$
$s - a = 32° 29' 59\frac{1}{2}''$	$\log \tan \frac{1}{2} A = 9.780862$
$s - b = 44° 18' 53\frac{1}{2}''$	$\frac{1}{2} A = 31° 07' 18''$
$s - c = 16° 15' 26\frac{1}{2}''$	$A = 62° 14' 36''$

The angles B and C are found in a similar manner.

Solving the same problem by the three formulas [58], we have

$\log \sin (s - a) = 9.730214$	$\log \tan \frac{1}{2} A = 9.780862$
$\log \sin (s - b) = 9.844229$	$\log \tan \frac{1}{2} B = 9.666847$
$\log \sin (s - c) = 9.447084$	$\log \tan \frac{1}{2} C = \underline{10.063992}$
$\text{colog} \sin s = \underline{0.000625}$	$\frac{1}{2} A = 31° 07' 18''$
$\log \tan^2 r = 19.022152$	$\frac{1}{2} B = 24° 54' 28''$
$\log \tan r = 9.511076$	$\frac{1}{2} C = \underline{49° 12' 22''}$
	$A = 62° 14' 36''$
	$B = 49° 48' 56''$
	$C = 98° 24' 44''$

NOTE.—To find $\log \tan \frac{1}{2} A$, we need not rewrite $\log \tan r$ and $\log \sin (s—a)$, but can subtract $\log \sin s - a$ from $\log \tan r$ as they stand in the first column; and similarly for $\tan \frac{1}{2} B$, and $\tan \frac{1}{2} C$.

2. Given $a = 120° 45' 28''$, $b = 62° 27' 40''$, $c = 108° 23' 40''$; find $A = 115° 44' 52''$, $B = 68° 20' 24''$, $C = 95° 57' 32''$.

3. Given $a = 135° 16' 40''$, $b = 110° 55' 30''$, $c = 86° 32' 16''$; find $A = 137° 38' 32''$, $B = 116° 34' 34''$, $C = 107° 06' 36''$.

4. Given $a = 25° 24' 23''$, $b = 48° 38' 28''$, $c = 76° 46' 36''$; show that this is impossible.

CASE VI.

206. *Given the three angles, A, B, and C.*

METHOD.—The sides may be found by the Formulas [59], [60], or [61]. The formulas for the tangent are usually preferred, since they require fewer logarithms and give accurate results in every part of the quadrant.

These formulas for the tangent may be put in a still more convenient form by substituting in each

$$\tan^2 R = -\cos S \sec (S - A) \sec (S - B) \sec (S - C),$$

which, when reduced, gives us

$$\left.\begin{aligned} \tan \tfrac{1}{2} a &= \tan R \div \sec (S - A). \\ \tan \tfrac{1}{2} b &= \tan R \div \sec (S - B). \\ \tan \tfrac{1}{2} c &= \tan R \div \sec (S - C). \end{aligned}\right\} \quad [66]$$

NOTE.—When only one side is required, use For's. [59, 60, 61]; when all the sides are required, use For's. [66].

EXERCISES XLVII.

1. Given $A = 106° \ 36'$, $B = 87° \ 45'$, $C = 96° \ 48'$; find a, b, and c.

SOLUTION.—Since the three sides are required, we solve by Formulas [66].

$$\begin{aligned} A &= 106° \ 36' \\ B &= 87° \ 45' \\ C &= \underline{96° \ 48'} \\ 2S &= 291° \ 09' \end{aligned}$$

$$\begin{aligned} \log \cos S &= 9.916384 \ (n) \\ \log \sec (S - A) &= 10.109344 \\ \log \sec (S - B) &= 10.273674 \\ \log \sec (S - C) &= \underline{10.181103} \\ \log \tan^2 R &= 40.480505 \\ \log \tan R &= 20.240252 \end{aligned}$$

$$\begin{aligned} S &= 145° \ 34' \ 30'' \\ S - A &= 38° \ 58' \ 30'' \\ S - B &= 57° \ 49' \ 30'' \\ S - C &= 48° \ 46' \ 30'' \end{aligned}$$

$$\begin{aligned} \log \tan \tfrac{1}{2} a &= 10.130908 \\ \log \tan \tfrac{1}{2} b &= 9.966578 \\ \log \tan \tfrac{1}{2} c &= \underline{10.059149} \end{aligned}$$

$$\begin{aligned} \tfrac{1}{2} a &= 53° \ 30' \ 26'' \\ \tfrac{1}{2} b &= 42° \ 47' \ 51'' \\ \tfrac{1}{2} c &= \underline{48° \ 53' \ 23''} \\ a &= 107° \ 00' \ 52'' \\ b &= 85° \ 35' \ 42'' \\ c &= 97° \ 46' \ 46'' \end{aligned}$$

NOTE.—To find log tan $\frac{1}{2}$ a, we need not rewrite log tan R and log sec $(S - A)$, but can subtract them as they stand in the first column; and similarly for log tan $\frac{1}{2}$ b and log tan $\frac{1}{2}$ c.

2. Given $A = 130° 46'$, $B = 110° 50'$, $C = 80° 30'$; find $a = 140° 32' 18''$, $b = 128° 20' 40''$, $c = 55° 51' 28''$.

3. Given $A = 60° 25' 40''$, $B = 87° 26' 32''$, $C = 60° 25' 40''$; find $a = 53° 36' 16''$, $b = 67° 36' 20''$, $c = 53° 36' 16''$.

4. Given $A = 90°$, $B = 90°$, $C = 90°$; find $a = 90°$, $b = 90°$, and $c = 90°$.

Solution by Means of a Perpendicular.

207. Oblique spherical triangles may be readily solved, also, by dividing them into right triangles and applying Napier's Rules.

Thus, let CD be a perpendicular drawn from C to the base AB.

1. Then, Rule II.

$$\cos a = \cos m \cos h.$$

Whence, $\cos h = \cos a \div \cos m$.

$$\cos b = \cos n \cos h.$$

Whence, $\cos h = \cos b \div \cos n$.

Whence, $\cos a : \cos m = \cos b : \cos n$.

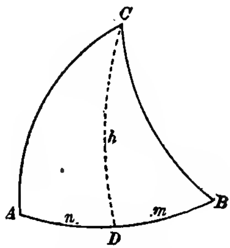

Fig. 57.

That is, *the cosines of the sides are proportional to the cosines of the segments of the base.*

2. Again, by Rule II.

$$\cos A = \cos h \sin ACD.$$

$$\cos B = \cos h \sin BCD.$$

Whence, $\cos A : \cos B = \sin ACD : \sin BCD$.

That is, *the cosines of the angles at the base are proportional to the sines of the segments of the vertical angle.*

3. Again, by Rule I.

$$\sin n = \tan h \cot A = \tan h \div \tan A.$$

$$\sin m = \tan h \cot B = \tan h \div \tan B.$$

Whence, $$\sin m : \sin n = \tan A : \tan B.$$

That is, *the sines of the segments of the base are inversely proportional to the tangents of the angles at the base.*

4. Again, by Rule I.

$$\cos ACD = \tan h \cot b.$$

$$\cos BCD = \tan h \cot a.$$

Whence, $$\cot a : \cot b = \cos BCD : \cos ACD.$$

That is, *the cotangents of the two sides are proportional to the cosines of the segments of the vertical angle.*

5. Again, by Art. 207, we have

$$\cos a : \cos b = \cos m : \cos n.$$

Whence,

$$\cos b + \cos a : \cos b - \cos a = \cos n + \cos m : \cos n - \cos m.$$

But, Art. 70,

$$\cos b + \cos a : \cos b - \cos a = \cot \tfrac{1}{2}(a+b) : \tan \tfrac{1}{2}(a-b).$$

$$\cos n + \cos m : \cos n - \cos m = \cot \tfrac{1}{2}(m+n) : \tan \tfrac{1}{2}(m-n).$$

Whence,

$$\cot \tfrac{1}{2}(a+b) : \cot \tfrac{1}{2}(m+n) = \tan \tfrac{1}{2}(a-b) : \tan \tfrac{1}{2}(m-n).$$

And since tangents are inversely proportioned to cotangents,

$$\cot \tfrac{1}{2}(a+b) : \cot \tfrac{1}{2}(m+n) = \tan \tfrac{1}{2}(m+n) : \tan \tfrac{1}{2}(a+b).$$

Whence,

$$\tan \tfrac{1}{2}(m+n) : \tan \tfrac{1}{2}(a+b) = \tan \tfrac{1}{2}(a-b) : \tan \tfrac{1}{2}(m-n).$$

That is, *the tangent of half the sum of the segments of the base is to the tangent of half the sum of the sides, as the tangent of half the difference of the sides is to the tangent of half the difference of the segments of the base.*

208. These five principles, derived immediately from Napier's Rules, enable us to solve every case of the oblique triangle. They are more easily remembered than the formulas previously used, and are preferred by some authors in solving these triangles.

EXERCISES XLVIII.

1. In the spherical triangle ABC, given $AC = 70° \ 30'$, $BC = 80° \ 36'$, and the angle $A = 35° \ 24'$; required the other parts.

SOLUTION.—Let ABC denote the triangle. Draw $CD \perp$ to AB.

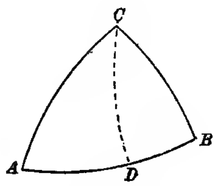

Fig. 58.

First, we have, Art. 183,

$$\sin BC : \sin AC = \sin A : \sin B.$$

Whence, $B = 33° \ 36' \ 23''$.

Then in triangle ACD, Rule I.,

$$\cos AC = \cot A \cot ACD.$$

Whence, $ACD = 76° \ 39'' \ 17''$.

Also in triangle BCD,

$$\cos BC = \cot B \cot BCD.$$

Whence, $BCD = 83° \ 48' \ 19''$.

Therefore, $ACB = 160° \ 27' \ 36''$.

Finally, we have

$$\sin A : \sin C = \sin BC : \sin AB.$$

Whence, $AB = 145° \ 16' \ 33''$.

2. In the spherical triangle ABC, given $A = 114° 36' 40''$, $B = 82° 27' 18''$, and $AC = 86° 20' 30''$; find $BC = 113° 45' 44''$, $AB = 82° 7' 18''$, and $ACB = 79° 44' 3''$.

3. In the spherical triangle ABC, given $AB = 72° 36'$, $AC = 40° 44'$, and $A = 54° 40'$; find $B = 41° 06' 02''$, $C = 105° 59' 16''$, and $BC = 54° 04' 32''$.

4. In the spherical triangle ABC, given $A = 108° 36' 45''$, $C = 40° 38' 28''$, and $AC = 56° 42' 22''$; find $BC = 76° 43' 35''$, $B = 54° 28' 40''$, and $AB = 41° 58' 57''$.

5. In the spherical triangle ABC, given $AB = 112° 25'$, $AC = 60° 20'$, $BC = 81° 10'$; find $A = 64° 46' 36''$, $B = 52° 42' 12''$, and $C = 122° 11' 06''$.

6. In the spherical triangle ABC, given $A = 106° 36'$, $B = 87° 45'$, $C = 96° 48'$; find $AB = 97° 46' 46''$, $BC = 107° 00' 55''$, $AC = 85° 35' 42''$.

NOTES.—The following suggestions will aid the student with the above examples.

1. In Ex. 1, since the value of AB is found from the sine, we determine its quadrant by Art. 160.

2. In Ex. 2, we first find BC by Art. 184; then by Rule I. find AD and BD; then take their sum and find ACB by Art. 184.

3. In Ex. 3, we first find AD by Rule I.; then find B by For. 3, Art. 207; then BC by Rule I., then BCD by Rule I., and then ACD by Rule I., from which we find C. The latter part of this solution prevents ambiguity.

4. In Ex. 4, first find ACD by Rule I., from which find BCD; then find BC by For. 4, Art. 207; then find B by Rule I.; then find AB by Art. 184.

5. In Ex. 5, first find AD and BD by Art. 184; then find A by Rule I.; then find B by Rule I.; then find C by Art. 188.

6. In Ex. 6, pass to the polar triangle; find its angles as in Ex. 5; the supplements of these angles will be the sides of the given triangle.

SECTION XV.

SUPPLEMENT.

Area of a Spherical Triangle.

209. We now proceed to show how to find the area of a spherical triangle.

I. *When the three angles, A, B, and C, are given.*

Let $R =$ the radius of the sphere.

$E =$ the spherical excess $= A + B + C - 180°$.

$S =$ the area of the triangle.

Then by Geometry, B. IX., Th. XXVII.,

$$S = \frac{\pi R^2 \times E}{180.}.$$

II. *When the three sides are given.*

210. Take $E = A + B + C - 180°$ as above; then

$$\tan \tfrac{1}{4} E = \frac{\sin \frac{1}{4}(A + B + C - 180)}{\cos \frac{1}{4}(A + B + C - 180)}$$

$$= \frac{\sin \frac{1}{2}(A + B) - \sin \frac{1}{2}(180 - C)}{\cos \frac{1}{2}(A + B) + \cos \frac{1}{2}(180 - C)}$$

Art. 70.

$$= \frac{\sin \frac{1}{2}(A + B) - \cos \frac{1}{2} C}{\cos \frac{1}{2}(A + B) + \sin \frac{1}{2} C}$$

$$= \frac{\cos \frac{1}{2}(a - b) - \cos \frac{1}{2} c}{\cos \frac{1}{2}(a + b) + \cos \frac{1}{2} c} \times \frac{\cos \frac{1}{2} C}{\sin \frac{1}{2} C} =$$

For. [62]

$$\frac{\sin \frac{1}{4}(c + a - b) \sin \frac{1}{4}(c + b - a)}{\sin \frac{1}{4}(a + b + c) \cos \frac{1}{4}(a + b - c)} \times \sqrt{\left\{\frac{\sin s \sin (s - c)}{\sin (s - a) \sin (s - b)}\right\}}.$$

For. [56, 57].

$$= \frac{\sin \frac{1}{2}(s - b) \sin \frac{1}{2}(s - a)}{\cos \frac{1}{2} s \cos \frac{1}{2}(s - c)} \times \sqrt{\left\{\frac{\sin s \sin (s - c)}{\sin (s - a) \sin (s - b)}\right\}}.$$

$$=\frac{\sqrt{\sin s}}{\cos\frac{1}{2}s}\times\frac{\sin\frac{1}{2}(s-b)}{\sqrt{\sin(s-b)}}\times\frac{\sqrt{\sin(s-c)}}{\cos\frac{1}{2}(s-c)}\times\frac{\sin\frac{1}{2}(s-a)}{\sqrt{\sin(s-a)}}.$$

Substituting for $\cos\frac{1}{2}s$, $\sin\frac{1}{2}(s-b)$, etc., their values of [26] and [25], and reducing, we have

$$\tan\tfrac{1}{4}E=\sqrt{\{\tan\tfrac{1}{2}s\tan\tfrac{1}{2}(s-a)\tan\tfrac{1}{2}(s-b)\tan\tfrac{1}{2}(s-c)}. \quad [67]$$

211. This elegant formula is known as L'Huiller's Theorem. By means of it the value of E may be found from the three sides, and then the area of the triangle may be found from Art. 209.

212. In a similar manner we can find Cagnoli's Theorem, which is

$$\sin\tfrac{1}{2}E=\frac{\sqrt{\{\sin s\sin(s-a)\sin(s-b)\sin(s-c)\}}}{2\cos\frac{1}{2}a\cos\frac{1}{2}b\cos\frac{1}{2}c}.$$

Circumscribed and Inscribed Circles.

I. *To find the radial arc of a circumscribed circle.*

213. Let O be the pole of the small circle circumscribed about the spherical triangle ABC. Draw the radial arcs OA, OB, and OC, and draw OD perpendicular to BC. The triangles OBC, AOC, and AOB are isosceles, and $BD=\frac{1}{2}a$. Denote the angles by A, B, C.

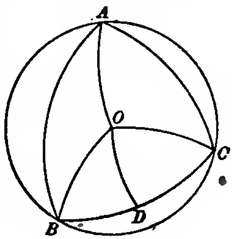

Fig. 59.

Denote the radial arc of the circumscribed circle by R.

Then in the right triangle BOD

we have $$\cos OBD=\cot R\tan\tfrac{1}{2}a,$$

Whence, $$\tan R=\frac{\tan\frac{1}{2}a}{\cos OBD}.$$

Now, $$OBD=B-ABO=B-BAO.$$

And, $$OBD=OCD=C-ACO=C-OAC.$$

Whence, $2\ OBD = B + C - (BAO + OAC)$

$$= B + C - A = 2\,(S - A),$$

hence, $OBD = S - A.$

Whence, $$\tan R = \frac{\tan \frac{1}{2} a}{\cos (S - A)}.$$

Similarly, $\tan R = \dfrac{\tan \frac{1}{2} b}{\cos (S - B)}$, and $\tan R = \dfrac{\tan \frac{1}{2} c}{\cos (S - C)}$.

Whence, $$\tan^3 R = \frac{\tan \frac{1}{2} a \tan \frac{1}{2} b \tan \frac{1}{2} c}{\cos (S - A) \cos (S - B) \cos (S - C)}.$$

The product of the three formulas, [61], gives

$$\tan^2 \tfrac{1}{2} a \tan^2 \tfrac{1}{2} b \tan^2 \tfrac{1}{2} c = -\frac{\cos^3 S}{\cos (S - A) \cos (S - B) \cos (S - C)}.$$

Substituting this in the value of $\tan^3 R$ and reducing

$$\tan R = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}. \qquad [68]$$

II. *To find the radial arc of an inscribed circle.*

214. Let O be the pole of the circle inscribed in the spherical triangle ABC. Draw the radial arcs OD, OE, and OF perpendicular to BC, AC, and AD respectively. Draw also the arc EGF.

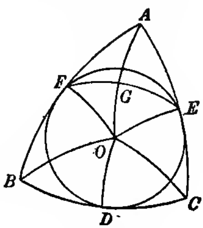

Fig. 60.

Now, since $OE = OF$, the triangle EOF is isosceles. Then

$\angle\, OFG = \angle\, OEG$ and $\angle\, GFA = \angle\, GEA.$

Hence, $\triangle\, FAE$ is isosceles; and $AF = AE.$

Draw the arc OG perpendicular to EF at its middle point; it will bisect the angle EOF, and will also pass through the point A and bisect the angle A. Similarly, the arcs OB and OC will bisect the angles B and C respectively. Denote the radial arc of the inscribed circle by r.

Then in the right triangle AOF

$$\sin AF = \tan r \cot \tfrac{1}{2} A.$$

Now, $\quad AF = AE;\ BF = BD;\ CE = CD.$

Also, $\quad AF = c - BF = c - BD.$

And, $\quad AE = b - CE = b - CD.$

Adding, $AF + AE = 2\,AF = b + c - (BD + CD).$

Or,
$$2\,AF = b + c - a.$$
$$= 2\,(s - a).$$
$$AF = s - a.$$

Substituting this value in $\sin AF$ and reducing,

We have $\quad \tan r = \sin (s - a) \tan \tfrac{1}{2} A.$

Similarly, $\quad \tan r = \sin (s - b) \tan \tfrac{1}{2} B.$

$\quad \tan r = \sin (s - c) \tan \tfrac{1}{2} C.$

The product of these three formulas gives

$$\tan^3 r = \sin (s - a) \sin (s - b) \sin (s - c) \tan \tfrac{1}{2} A \tan \tfrac{1}{2} B \tan \tfrac{1}{2} C.$$

Finding the value of $\tan \tfrac{1}{2} A \tan \tfrac{1}{2} B \tan \tfrac{1}{2} C$ from For's. [58], and substituting and reducing, we have

$$\tan r = \sqrt{\left\{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}\right\}} \qquad [69]$$

EXERCISES XLIX.

1. Find the area of a spherical triangle whose angles are 60° 30′, 70° 40′, and 80° 50′.

2. Find the area of a spherical triangle whose sides are 60° 30′, 70° 40′ and 80° 50′.

3. Find the radius of the circumscribed circle in the triangle of Exercise 1.

4. Find the radius of the inscribed circle in the triangle of Exercise 2.

Miscellaneous Exercises.

1. In any spherical triangle, if $A = a$, show that $B = b$, and $C = c$, or that they are respectively supplementary.

2. When does the polar triangle coincide with the primitive triangle?

3. If $B = A + C$ and D is the middle point of b, prove that $b = 2\,AD$.

4. If D is the middle point of c, prove that

$$\cos b + \cos a = 2 \cos \tfrac{1}{2} c \cos CD.$$

5. If $b + c = \pi$, prove that $\sin 2\,B + \sin 2\,C = 0$.

6. In an equilateral spherical triangle, prove that $2 \cos \frac{1}{2} a \sin \frac{1}{2} A = 1$.

7. In an equilateral spherical triangle, prove that $\tan^2 \frac{1}{2} a = 1 - 2 \cos A$.

8. In an equilateral spherical triangle, prove that $\sec A = 1 + \sec a$.

9. If $b = c = 2a$, prove that $\csc \frac{1}{2} A = 4 \cos a \cos \frac{1}{2} a$.

Right Spherical Triangle.

1. Derive two rules similar to those of Napier for the direct solution of quadrantal triangles.

If ABC is a right triangle, C being the right angle, then

2. Prove $\sin^2 \frac{1}{2} c = \sin^2 \frac{1}{2} a \cos^2 \frac{1}{2} b + \cos^2 \frac{1}{2} a \sin^2 \frac{1}{2} b$.

3. Prove $\tan \frac{1}{2} (c + a) \tan \frac{1}{2} (c - a) = \tan^2 \frac{1}{2} b$.

4. Prove $\sin (c - b) = \tan^2 \frac{1}{2} A \sin (c + b)$.

5. Prove $\sin a \tan \frac{1}{2} A - \sin b \tan \frac{1}{2} B = \sin (a - b)$.

6. Prove

$$\sin (c - a) = \sin b \cos a \tan \tfrac{1}{2} B.$$
$$\sin (c - a) = \tan b \cos c \tan \tfrac{1}{2} B.$$

7. Prove $\sin(c+a) = \sin b \cos a \cot \frac{1}{2} B.$

$\sin(c+a) = \tan b \cos c \cot \frac{1}{2} B.$

8. In a right spherical triangle, C the right angle, if D is the middle point of AB, prove that

$$\sin^2 a + \sin^2 b = 4 \cos^2 \tfrac{1}{2} c \sin^2 CD.$$

9. In a right triangle, if d is the length of the arc from C perpendicular to the hypotenuse, prove that

$$\cot^2 a + \cot^2 b = \cot^2 d.$$

10. If ABC is a right spherical triangle, A not being the right angle, prove that if $A = a$, then b and c are quadrants.

In a right triangle, C being the right angle, prove

11. $\tan^2 \frac{1}{2} A = \dfrac{\sin(c-b)}{\sin(c+b)}.$

13. $\tan^2(45° - \frac{1}{2} A) = \dfrac{\tan \frac{1}{2}(c-a)}{\tan \frac{1}{2}(c+a)}.$

12. $\tan^2 \frac{1}{2} c = -\dfrac{\cos(A+B)}{\cos(A-B)}.$

14. $\dfrac{\cos a}{\cos b} = \dfrac{\sin 2A}{\sin 2B}.$

Oblique Spherical Triangles.

1. If the area of an equilateral triangle is one-fourth of the area of the sphere, what are its sides and angles?

2. In a spherical triangle, if $c = 90°$ and d denotes the perpendicular from C to c, then $\cos^2 d = \cos^2 a + \cos^2 b$.

3. In a spherical triangle, if $A = B = 2C$; then

$$8 \sin(a + \tfrac{1}{2} c) \sin^2 \tfrac{1}{2} c \cos \tfrac{1}{2} c = \sin^3 a.$$

4. In a spherical triangle, if $A = B = 2C$; then

$$8 \sin^2 \tfrac{1}{2} C (\cos s + \sin \tfrac{1}{2} C) \cos \tfrac{1}{2} c = \cos a.$$

5. In any equilateral triangle, R and r denoting respectively the radii of circumscribed and inscribed circles, prove $\tan R = 2 \tan r$.

6. In any spherical triangle, E denoting the spherical excess, prove

$$\sin \tfrac{1}{2} E = \sin C \sin \tfrac{1}{2} a \sin \tfrac{1}{2} b \sec \tfrac{1}{2} c.$$

7. In any spherical triangle, E denoting the spherical excess, prove

$$\cos \tfrac{1}{2} E = \{\cos \tfrac{1}{2} a \cos \tfrac{1}{2} b + \sin \tfrac{1}{2} a \sin \tfrac{1}{2} b \cos C\} \sec \tfrac{1}{2} c.$$

8. If the angle C of a spherical triangle is a right angle, prove

$$\sin \tfrac{1}{2} E = \sin \tfrac{1}{2} a \sin \tfrac{1}{2} b \sec \tfrac{1}{2} c;$$
$$\cos \tfrac{1}{2} E = \cos \tfrac{1}{2} a \cos \tfrac{1}{2} b \sec \tfrac{1}{2} c.$$

9. If the angle C is a right angle, prove that

$$\frac{\sin^2 c}{\cos c} \cos E = \frac{\sin^2 a}{\cos a} + \frac{\sin^2 b}{\cos b}.$$

10. If $a = b$ and $C = \frac{\pi}{2}$, prove that $E = \frac{\sin^2 a}{2 \cos a}$.

11. If the angles of a spherical triangle are together equal to four right angles, prove

$$\cos^2 \tfrac{1}{2} a + \cos^2 \tfrac{1}{2} b + \cos^2 \tfrac{1}{2} c = 1.$$

12. If ABC is an equilateral spherical triangle, P the pole of the circumscribed circle, and Q any point on the sphere, prove that

$$\cos AQ + \cos BQ + \cos CQ = 3 \cos R \cos PQ.$$

13. Find the surface of an equilateral and equiangular spherical polygon of n sides, and determine the value of each of the angles when the surface equals one-half the surface of the sphere.

A TABLE

OF

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0·000000	26	1·414973	51	1·707570	76	1·880814
2	0·301030	27	1·431364	52	1·716003	77	1·886491
3	0·477121	28	1·447158	53	1·724276	78	1·892095
4	0·602060	29	1·462398	54	1·732394	79	1·897627
5	0·698970	30	1·477121	55	1·740363	80	1·903090
6	0·778151	31	1·491362	56	1·748188	81	1·908485
7	0·845098	32	1·505150	57	1·755875	82	1·913814
8	0·903090	33	1·518514	58	1·763428	83	1·919078
9	0·954243	34	1·531479	59	1·770852	84	1·924279
10	1·000000	35	1·544068	60	1·778151	85	1·929419
11	1·041393	36	1·556303	61	1·785330	86	1·934498
12	1·079181	37	1·568202	62	1·792392	87	1·939519
13	1·113943	38	1·579784	63	1·799341	88	1·944483
14	1·146128	39	1·591065	64	1·806181	89	1·949390
15	1·176091	40	1·602060	65	1·812913	90	1·954243
16	1·204120	41	1·612784	66	1·819544	91	1·959041
17	1·230449	42	1·623249	67	1·826075	92	1·963788
18	1·255273	43	1·633468	68	1·832509	93	1·968483
19	1·278754	44	1·643453	69	1·838849	94	1·973128
20	1·301030	45	1·653213	70	1·845098	95	1·977724
21	1·322219	46	1·662758	71	1·851258	96	1·982271
22	1·342423	47	1·672098	72	1·857333	97	1·986772
23	1·361728	48	1·681241	73	1·863323	98	1·991226
24	1·380211	49	1·690196	74	1·869232	99	1·995635
25	1·397940	50	1·698970	75	1·875061	100	2·000000

REMARK.—In the following table, in the nine right-hand columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's, to catch the eye, and to indicate that from thence the two figures of the Logarithm to be taken from the second column, stand in the next line below.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	•300	•724	1147	1570	1993	2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	•361	•775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6042	7350	7757	8164	8571	8978	408
107	9384	9789	•195	•600	1004	1408	1812	2216	2619	3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	•207	•602	•998	396
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	•380	•766	1153	1538	1924	2309	2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	•320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	8186	8557	8928	9298	9668	••38	•407	•776	1145	1514	369
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	•266	•626	•987	1347	1707	2067	2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	•258	•611	•963	1315	1667	2018	2370	2721	3071	351
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	••26	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	•253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	•245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	••12	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	9879	•194	•508	•822	1136	1450	1763	2076	2389	2702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	•142	•449	•756	1063	1370	1676	1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	•168	•469	•769	1068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	•126	•413	•699	•985	1272	1558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	••51	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5899	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	••29	•303	•577	•850	1124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
160	204120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	••51	•319	•586	•853	1121	1388	1654	1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	•193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	••50	•300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	•176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	•213	•446	•679	•912	1144	1377	1609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8206	8525	229
190	278754	8982	9211	9439	9667	9895	•123	•351	•578	•806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	•161	•378	•595	•813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	••56	•268	•481	•693	•906	1118	1330	1542	212
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	110
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	•••8	•211	203
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	••47	•246	199
219	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
220	342423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	••54	194
224	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	••25	•215	•404	•593	•783	•972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	•143	•328	•513	•698	•883	185
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	••30	181
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	9166	9343	9520	9698	9875	••51	•228	•405	•582	•759	177
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	••20	•192	•365	•538	•711	•883	1056	1228	173
252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	•102	•271	•440	•609	•777	•946	1114	1283	1451	169
258	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	•121	•286	•451	•616	•781	•945	1110	1275	1439	165
264	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	9752	9914	••75	•236	•398	•559	•720	•881	1042	1203	161
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6798	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	9333	9491	9648	9806	9964	•122	•279	•437	•594	•752	158
276	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
N.	0	1	2	3	4	5	6	7	8	9	D

N.	0	1	2	3	4	5	6	7	8	9	D.
280	447158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	••95	154
282	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	9392	9543	9694	9845	9995	•146	•296	•447	•597	•748	151
289	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	462398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	9822	9969	•116	•263	•410	•557	•704	•851	•998	1145	147
296	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958	••99	•239	•380	•520	•661	•801	•941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	••99	•236	•374	•511	•648	•785	•922	137
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	9203	9337	9471	9606	9740	9874	•••9	•143	•277	•411	134
324	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4414	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959	••90	•221	•353	•484	•615	•745	•876	1007	131
332	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	••72	128
339	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	9076	9202	9327	9452	9578	9703	9829	9954	•079	•204	125
347	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	•106	123
355	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	•026	•146	•265	•385	•504	•624	•743	•863	•982	119
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	•076	•193	•309	•426	117
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	9898	•012	•126	•241	•355	•469	•583	•697	•811	114
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	9950	•061	•173	•284	•396	•507	•619	•730	•842	•953	112
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	•101	•210	•319	•428	•537	•646	•755	•864	109
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
400	602060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914	••21	•128	•234	•341	•447	•554	107
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	••32	104
417	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919	••21	•123	•224	•326	102
427	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	3569	3670	3771	3872	3973	4074	4175	4276	4376	100
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	100
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	99
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
436	9486	9586	9686	9785	9885	9984	••84	•183	•283	•382	99
437	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	••16	•113	•210	97
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	••11	•106	•201	•296	•391	•486	•581	•676	•771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	••60	•153	93
468	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	••63	•154	•245	91
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	681241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	••19	•107	89
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	9838	9924	••11	••98	•184	•271	•358	•444	•531	•617	87
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	••33	85
513	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
520	716003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	••77	83
525	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	••55	•136	•217	•298	•378	•459	•540	•621	•702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	••47	•126	•205	•284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	••45	•123	•200	•277	•354	•431	77
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	~~3583~~	~~3660~~	~~3736~~	~~3813~~	~~3889~~	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	••45	•121	•196	•272	•347	75
576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
580	763428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	••42	74
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	770852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	••29	•101	•173	•245	72
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	•••4	••74	•144	•215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
N.	0	1	2	3	4	5	6	7	8	9	D

N.	0	1	2	3	4	5	6	7	8	9	D.
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	••31	••98	•165	67
646	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	9610	9676	9741	9807	9873	9939	•••4	••70	•136	66
661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9947	••11	••75	•139	•204	•268	•332	•396	•460	•525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	••43	63
692	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
700	845098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	9739	9799	9859	9918	9978	•038	•098	•158	•218	•278	60
725	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	9818	9877	9935	9994	•053	•111	•170	•228	•287	•345	59
742	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	2727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	•013	•070	•127	•185	57
759	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
760	880814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	9862	9918	9974	••30	••86	•141	•197	•253	•309	•365	56
777	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	••39	••94	•149	•203	•258	•312	55
795	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2320	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9610	9663	9716	9770	9823	9877	9930	9984	••37	53
813	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
820	913814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	••19	••71	52
832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	••32	••83	•134	•185	•236	•287	•338	•389	51
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3040	3009	3148	3198	3247	3297	3346	3396	3445	59
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	59
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	59
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	••24	••73	•121	•170	•219	•267	•316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	••42	••90	•138	•185	•233	•280	•328	•376	•423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	••21	••68	•114	•161	•207	•254	•300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	••28	••72	•117	•161	•206	•250	•294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE

OF

LOGARITHMIC

SINES AND TANGENTS

FOR EVERY

DEGREE AND MINUTE

OF THE QUADRANT.

REMARK. The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	0·000000		10·000000		0·000000		Infinite.	60
1	6·463726	5017·17	000000	·00	6·463726	5017·17	13·536274	59
2	764756	2934·85	000000	·00	764756	2934·83	235244	58
3	940847	2082·31	000000	·00	940847	2082·31	059153	57
4	7·065786	1615·17	000000	·00	7·065786	1615·17	12·934214	56
5	162696	1319·68	000000	·00	162696	1319·69	837304	55
6	241877	1115·75	9·999999	·01	241878	1115·78	758122	54
7	308824	966·53	999999	·01	308825	996·53	691175	53
8	366816	852·54	999999	·01	366817	852·54	633183	52
9	417968	762·63	999999	·01	417970	762·63	582030	51
10	463725	689·88	999998	·01	463727	689·88	536273	50
11	7·505118	629·81	9·999998	·01	7·505120	629·81	12·494880	49
12	542906	579·36	999997	·01	542909	579·33	457091	48
13	577668	536·41	999997	·01	577672	536·42	422328	47
14	609853	499·38	999996	·01	609857	499·39	390143	46
15	639816	467·14	999996	·01	639820	467·15	360180	45
16	667845	438·81	999995	·01	667849	438·82	332151	44
17	694173	413·72	999995	·01	694179	413·73	305821	43
18	718997	391·35	999994	·01	719004	391·36	280997	42
19	742477	371·27	999993	·01	742484	371·28	257516	41
20	764754	353·15	999993	·01	764761	351·36	235239	40
21	7·785943	336·72	9·999992	·01	7·785951	336·73	12·214049	39
22	806146	321·75	999991	·01	806155	321·76	193845	38
23	825451	308·05	999990	·01	825460	308·06	174540	37
24	843934	295·47	999989	·02	843944	295·49	156056	36
25	861662	283·88	999988	·02	861674	283·90	138326	35
26	878695	273·17	999988	·02	878708	273·18	121292	34
27	895085	263·23	999987	·02	895099	263·25	104901	33
28	910879	253·99	999986	·02	910894	254·01	089106	32
29	926119	245·38	999985	·02	926134	245·40	073866	31
30	940842	237·33	999983	·02	940858	237·35	059142	30
31	7·955082	229·80	9·999982	·02	7·955100	229·81	12·044900	29
32	968870	222·73	999981	·02	968889	222·75	031111	28
33	982233	216·08	999980	·02	982253	216·10	017747	27
34	995198	209·81	999979	·02	995219	209·83	004781	26
35	8·007787	203·90	999977	·02	8·007809	203·92	11·992191	25
36	020021	198·31	999976	·02	020045	198·33	979955	24
37	031919	193·02	999975	·02	031945	193·05	968055	23
38	043501	188·01	999973	·02	043527	188·03	956473	22
39	054781	183·25	999972	·02	054809	183·27	945191	21
40	065776	178·72	999971	·02	065806	178·74	934194	20
41	8·076500	174·41	9·999969	·02	8·076531	174·44	11·923469	19
42	086965	170·31	999968	·02	086997	170·34	913003	18
43	097183	166·39	999966	·02	097217	166·42	902783	17
44	107167	162·65	999964	·03	107202	162·68	892797	16
45	116926	159·08	999963	·03	116963	159·10	883037	15
46	126471	155·66	999961	·03	126510	155·68	873490	14
47	135810	152·38	999959	·03	135851	152·41	864149	13
48	144953	149·24	999958	·03	144996	149·27	855004	12
49	153907	146·22	999956	·03	153952	146·27	846048	11
50	162681	143·33	999954	·03	162727	143·36	837273	10
51	8·171280	140·54	9·999952	·03	8·171328	140·57	11·828672	9
52	179713	137·86	999950	·03	179763	137·90	820237	8
53	187985	135·29	999948	·03	188036	135·32	811964	7
54	196102	132·80	999946	·03	196156	132·84	803844	6
55	204070	130·41	999944	·03	204126	130·44	795874	5
56	211895	128·10	999942	·04	211953	128·14	788047	4
57	219581	125·87	999940	·04	219641	125·90	780359	3
58	227134	123·72	999938	·04	227195	123·76	772805	2
59	234557	121·64	999936	·04	234621	121·68	765379	1
60	241855	119·63	999934	·04	241921	119·67	758079	0
	Cosine	D.	Sine	89°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	249033	117.68	999932	.04	249192	117.72	750808	59
2	256094	115.80	999929	.04	256165	115.84	743835	58
3	263042	113.98	999927	.04	263115	114.02	736885	57
4	269881	112.21	999925	.04	269956	112.25	730044	56
5	276614	110.50	999922	.04	276691	110.54	723309	55
6	283243	108.83	999920	.04	283323	108.87	716677	54
7	289773	107.21	999918	.04	289856	107.26	710144	53
8	296207	105.65	999915	.04	296292	105.70	703708	52
9	302546	104.13	999913	.04	302634	104.18	697366	51
10	308794	102.66	999910	.04	308884	102.70	691116	50
11	8.314904	101.22	9.999907	.04	8.315046	101.26	11.684954	49
12	321027	99.82	999905	.04	321122	99.87	678878	48
13	327016	98.47	999902	.04	327114	98.51	672886	47
14	332924	97.14	999899	.05	333025	97.19	666975	46
15	338753	95.86	999897	.05	338856	95.90	661144	45
16	344504	94.60	999894	.05	344610	94.65	655390	44
17	350181	93.38	999891	.05	350289	93.43	649711	43
18	355783	92.19	999888	.05	355895	92.24	644105	42
19	361315	91.03	999885	.05	361430	91.08	638570	41
20	366777	89.90	999882	.05	366895	89.95	633105	40
21	8.372171	88.80	9.999879	.05	8.372292	88.85	11.627708	39
22	377499	87.72	999876	.05	377622	87.77	622378	38
23	382762	86.67	999873	.05	382889	86.72	617111	37
24	387962	85.64	999870	.05	388092	85.70	611908	36
25	393101	84.64	999867	.05	393234	84.70	606766	35
26	398179	83.66	999864	.05	398315	83.71	601685	34
27	403199	82.71	999861	.05	403338	82.76	596662	33
28	408161	81.77	999858	.05	408304	81.82	591696	32
29	413068	80.86	999854	.05	413213	80.91	586787	31
30	417919	79.96	999851	.06	418068	80.02	581932	30
31	8.422717	79.09	9.999848	.06	8.422869	79.14	11.577131	29
32	427462	78.23	999844	.06	427618	78.30	572382	28
33	432156	77.40	999841	.06	432315	77.45	567685	27
34	436800	76.57	999838	.06	436962	76.63	563038	26
35	441394	75.77	999834	.06	441560	75.83	558440	25
36	445941	74.99	999831	.06	446110	75.05	553890	24
37	450440	74.22	999827	.06	450613	74.28	549387	23
38	454893	73.46	999823	.06	455070	73.52	544930	22
39	459301	72.73	999820	.06	459481	72.79	540519	21
40	463665	72.00	999816	.06	463849	72.06	536151	20
41	8.467985	71.29	9.999812	.06	8.468172	71.35	11.531828	19
42	472263	70.60	999809	.06	472454	70.66	527546	18
43	476498	69.91	999805	.06	476693	69.98	523307	17
44	480693	69.24	999801	.06	480892	69.31	519108	16
45	484848	68.59	999797	.07	485050	68.65	514950	15
46	488963	67.94	999793	.07	489170	68.01	510830	14
47	493040	67.31	999790	.07	493250	67.38	506750	13
48	497078	66.69	999786	.07	497293	66.76	502707	12
49	501080	66.08	999782	.07	501298	66.15	498702	11
50	505045	65.48	999778	.07	505267	65.55	494733	10
51	8.508974	64.89	9.999774	.07	8.509200	64.96	11.490800	9
52	512867	64.31	999769	.07	513098	64.39	486902	8
53	516726	63.75	999765	.07	516961	63.82	483039	7
54	520551	63.19	999761	.07	520790	63.26	479210	6
55	524343	62.64	999757	.07	524586	62.72	475414	5
56	528102	62.11	999753	.07	528349	62.18	471651	4
57	531828	61.58	999748	.07	532080	61.65	467920	3
58	535523	61.06	999744	.07	535779	61.13	464221	2
59	539186	60.55	999740	.07	539447	60.62	460553	1
60	542819	60.04	999735	.07	543084	60.12	456916	0
	Cosine	D.	Sine	88°	Cotang.	D.	Tang	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	546422	59.55	999731	.07	546691	59.62	453309	59
2	549995	59.06	999726	.07	550268	59.14	449732	58
3	553539	58.58	999722	.08	553817	58.66	446183	57
4	557054	58.11	999717	.08	557336	58.19	442664	56
5	560540	57.65	999713	.08	560828	57.73	439172	55
6	563999	57.19	999708	.08	564291	57.27	435709	54
7	567431	56.74	999704	.08	567727	56.82	432273	53
8	570836	56.30	999699	.08	571137	56.38	428863	52
9	574214	55.87	999694	.08	574520	55.95	425480	51
10	577566	55.44	999689	.08	577877	55.52	422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	584193	54.60	999680	.08	584514	54.68	415486	48
13	587469	54.19	999675	.08	587795	54.27	412205	47
14	590721	53.79	999670	.08	591051	53.87	408949	46
15	593948	53.39	999665	.08	594283	53.47	405717	45
16	597152	53.00	999660	.08	597492	53.08	402508	44
17	600332	52.61	999655	.08	600677	52.70	399323	43
18	603489	52.23	999650	.08	603839	52.32	396161	42
19	606623	51.86	999645	.09	606978	51.94	393022	41
20	609734	51.49	999640	.09	610094	51.58	389906	40
21	8.612823	51.12	9.999635	.09	8.613189	51.21	11.386811	39
22	615891	50.76	999629	.09	616262	50.85	383738	38
23	618937	50.41	999624	.09	619313	50.50	380687	37
24	621962	50.06	999619	.09	622343	50.15	377657	36
25	624965	49.72	999614	.09	625352	49.81	374648	35
26	627948	49.38	999608	.09	628340	49.47	371660	34
27	630911	49.04	999603	.09	631308	49.13	368692	33
28	633854	48.71	999597	.09	634256	48.80	365744	32
29	636776	48.39	999592	.09	637184	48.48	362816	31
30	639680	48.06	999586	.09	640093	48.16	359907	30
31	8.642563	47.75	9.999581	.09	8.642982	47.84	11.357018	29
32	645428	47.43	999575	.09	645853	47.53	354147	28
33	648274	47.12	999570	.09	648704	47.22	351296	27
34	651102	46.82	999564	.09	651537	46.91	348463	26
35	653911	46.52	999558	.10	654352	46.61	345648	25
36	656702	46.22	999553	.10	657149	46.31	342851	24
37	659475	45.92	999547	.10	659928	46.02	340072	23
38	662230	45.63	999541	.10	662689	45.73	337311	22
39	664968	45.35	999535	.10	665433	45.44	334567	21
40	667689	45.06	999529	.10	668160	45.26	331840	20
41	8.670393	44.79	9.999524	.10	8.670870	44.88	11.329130	19
42	673080	44.51	999518	.10	673563	44.61	326437	18
43	675751	44.24	999512	.10	676239	44.34	323761	17
44	678405	43.97	999506	.10	678900	44.17	321100	16
45	681043	43.70	999500	.10	681544	43.80	318456	15
46	683665	43.44	999493	.10	684172	43.54	315828	14
47	686272	43.18	999487	.10	686784	43.28	313216	13
48	688863	42.92	999481	.10	689381	43.03	310619	12
49	691438	42.67	999475	.10	691963	42.77	308037	11
50	693998	42.42	999469	.10	694529	42.52	305471	10
51	8.696543	42.17	9.999463	.11	8.697081	42.28	11.302919	9
52	699073	41.92	999456	.11	699617	42.03	300383	8
53	701589	41.68	999450	.11	702139	41.79	297861	7
54	704090	41.44	999443	.11	704646	41.55	295354	6
55	706577	41.21	999437	.11	707140	41.32	292860	5
56	709049	40.97	999431	.11	709618	41.08	290382	4
57	711507	40.74	999424	.11	712083	40.85	287917	3
58	713952	40.51	999418	.11	714534	40.62	285465	2
59	716383	40.29	999411	.11	716972	40.40	283028	1
60	718800	40.06	999404	.11	719396	40.17	280604	0
	Cosine	D.	Sine	87°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·718800	40·06	9·999404	·11	8·719396	40·17	11·280604	60
1	721204	39·84	999398	·11	721806	39·95	278194	59
2	723595	39·62	999391	·11	724204	39·74	275796	58
3	725972	39·41	999384	·11	726588	39·52	273412	57
4	728337	39·19	999378	·11	728959	39·30	271041	56
5	730688	38·98	999371	·11	731317	39·09	268683	55
6	733027	38·77	999364	·12	733663	38·89	266337	54
7	735354	38·57	999357	·12	735996	38·68	264004	53
8	737667	38·36	999350	·12	738317	38·48	261683	52
9	739969	38·16	999343	·12	740626	38·27	259374	51
10	742259	37·96	999336	·12	742922	38·07	257078	50
11	8·744536	37·76	9·999329	·12	8·745207	37·87	11·254793	49
12	746802	37·56	999322	·12	747479	37·68	252521	48
13	749055	37·37	999315	·12	749740	37·49	250260	47
14	751297	37·17	999308	·12	751989	37·29	248011	46
15	753528	36·98	999301	·12	754227	37·10	245773	45
16	755747	36·79	999294	·12	756453	36·92	243547	44
17	757955	36·61	999286	·12	758668	36·73	241332	43
18	760151	36·42	999279	·12	760872	36·55	239128	42
19	762337	36·24	999272	·12	763065	36·36	236935	41
20	764511	36·06	999265	·12	765246	36·18	234754	40
21	8·766675	35·88	9·999257	·12	8·767417	36·00	11·232583	39
22	768828	35·70	999250	·13	769578	35·83	230422	38
23	770970	35·53	999242	·13	771727	35·65	228273	37
24	773101	35·35	999235	·13	773866	35·48	226134	36
25	775223	35·18	999227	·13	775995	35·31	224005	35
26	777333	35·01	999220	·13	778114	35·14	221886	34
27	779434	34·84	999212	·13	780222	34·97	219778	33
28	781524	34·67	999205	·13	782320	34·80	217680	32
29	783605	34·51	999197	·13	784408	34·64	215592	31
30	785675	34·31	999189	·13	786486	34·47	213514	30
31	8·787736	34·18	9·999181	·13	8·788554	34·31	11·211446	29
32	789787	34·02	999174	·13	790613	34·15	209387	28
33	791828	33·86	999166	·13	792662	33·99	207338	27
34	793859	33·70	999158	·13	794701	33·83	205299	26
35	795881	33·54	999150	·13	796731	33·68	203269	25
36	797894	33·39	999142	·13	798752	33·52	201248	24
37	799897	33·23	999134	·13	800763	33·37	199237	23
38	801892	33·08	999126	·13	802765	33·22	197235	22
39	803876	32·93	999118	·13	804758	33·07	195242	21
40	805852	32·78	999110	·13	806742	32·92	193258	20
41	8·807819	32·63	9·999102	·13	8·808717	32·78	11·191283	19
42	809777	32·49	999094	·14	810683	32·62	189317	18
43	811726	32·34	999086	·14	812641	32·48	187359	17
44	813667	32·19	999077	·14	814589	32·33	185411	16
45	815599	32·05	999069	·14	816529	32·19	183471	15
46	817522	31·91	999061	·14	818461	32·05	181539	14
47	819436	31·77	999053	·14	820384	31·91	179616	13
48	821343	31·63	999044	·14	822298	31·77	177702	12
49	823240	31·49	999036	·14	824205	31·63	175795	11
50	825130	31·35	999027	·14	826103	31·50	173897	10
51	8·827011	31·22	9·999019	·14	8·827992	31·36	11·172008	9
52	828884	31·08	999010	·14	829874	31·23	170126	8
53	830749	30·95	999002	·14	831748	31·10	168252	7
54	832607	30·82	998993	·14	833613	30·96	166387	6
55	834456	30·69	998984	·14	835471	30·83	164529	5
56	836297	30·56	998976	·14	837321	30·70	162679	4
57	838130	30·43	998967	·15	839163	30·57	160837	3
58	839956	30·30	998958	·15	840998	30·45	159002	2
59	841774	30·17	998950	·15	842825	30·32	157175	1
60	843585	30·00	998941	·15	844644	30·19	155356	0
	Cosine	D.	Sine	86°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	′
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	845387	29.92	998932	.15	846455	30.07	153545	59
2	847183	29.80	998923	.15	848260	29.95	151740	58
3	848971	29.67	998914	.15	850057	29.82	149943	57
4	850751	29.55	998905	.15	851846	29.70	148154	56
5	852525	29.43	998896	.15	853628	29.58	146372	55
6	854291	29.31	998887	.15	855403	29.46	144597	54
7	856049	29.19	998878	.15	857171	29.35	142829	53
8	857801	29.07	998869	.15	858932	29.23	141068	52
9	859546	28.96	998860	.15	860686	29.11	139314	51
10	861283	28.84	998851	.15	862433	29.00	137567	50
11	8.863014	28.73	9.998841	.15	8.864173	28.88	11.135827	49
12	864738	28.61	998832	.15	865906	28.77	134094	48
13	866455	28.50	998823	.16	867632	28.66	132368	47
14	868165	28.39	998813	.16	869351	28.54	130649	46
15	869868	28.28	998804	.16	871064	28.43	128936	45
16	871565	28.17	998795	.16	872770	28.32	127230	44
17	873255	28.06	998785	.16	874469	28.21	125531	43
18	874938	27.95	998776	.16	876162	28.11	123838	42
19	876615	27.86	998766	.16	877849	28.00	122151	41
20	878285	27.73	998757	.16	879529	27.89	120471	40
21	8.879949	27.63	9.998747	.16	8.881202	27.79	11.118798	39
22	881607	27.52	998738	.16	882869	27.68	117131	38
23	883258	27.42	998728	.16	884530	27.58	115470	37
24	884903	27.31	998718	.16	886185	27.47	113815	36
25	886542	27.21	998708	.16	887833	27.37	112167	35
26	888174	27.11	998699	.16	889476	27.27	110524	34
27	889801	27.00	998689	.16	891112	27.17	108888	33
28	891421	26.90	998679	.16	892742	27.07	107258	32
29	893035	26.80	998669	.17	894366	26.97	105634	31
30	894643	26.70	998659	.17	895984	26.87	104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.77	11.102404	29
32	897842	26.51	998639	.17	899203	26.67	100797	28
33	899432	26.41	998629	.17	900803	26.58	099197	27
34	901017	26.31	998619	.17	902398	26.48	097602	26
35	902596	26.22	998609	.17	903987	26.38	096013	25
36	904169	26.12	998599	.17	905570	26.29	094430	24
37	905736	26.03	998589	.17	907147	26.20	092853	23
38	907297	25.93	998578	.17	908719	26.10	091281	22
39	908853	25.84	998568	.17	910285	26.01	089715	21
40	910404	25.75	998558	.17	911846	25.92	088154	20
41	8.911949	25.66	9.998548	.17	8.913401	25.83	11.086599	19
42	913488	25.56	998537	.17	914951	25.74	085049	18
43	915022	25.47	998527	.17	916495	25.65	083505	17
44	916550	25.38	998516	.18	918034	25.56	081966	16
45	918073	25.29	998506	.18	919568	25.47	080432	15
46	919591	25.20	998495	.18	921096	25.38	078904	14
47	921103	25.12	998485	.18	922619	25.30	077381	13
48	922610	25.03	998474	.18	924136	25.21	075864	12
49	924112	24.94	998464	.18	925649	25.12	074351	11
50	925609	24.86	998453	.18	927156	25.03	072844	10
51	8.927100	24.77	9.998442	.18	8.928658	24.95	11.071342	9
52	928587	24.69	998431	.18	930155	24.86	069845	8
53	930068	24.60	998421	.18	931647	24.78	068353	7
54	931544	24.52	998410	.18	933134	24.70	066866	6
55	933015	24.43	998399	.18	934616	24.61	065384	5
56	934481	24.35	998388	.18	936093	24.53	063907	4
57	935942	24.27	998377	.18	937565	24.45	062435	3
58	937398	24.19	998366	.18	939032	24.37	060968	2
59	938850	24.11	998355	.18	940494	24.30	059506	1
60	940296	24.03	998344	.18	941952	24.21	058048	0
′	Cosine	D.	Sine	85°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8·940296	24·03	9·998344	·19	8·941952	24·21	11·058048	60
1	941738	23·94	998333	·19	943404	24·13	056596	59
2	943174	23·87	998322	·19	944852	24·05	055148	58
3	944606	23·79	998311	·19	946295	23·97	053705	57
4	946034	23·71	998300	·19	947734	23·90	052266	56
5	947456	23·63	998289	·19	949168	23·82	050832	55
6	948874	23·55	998277	·19	950597	23·74	049403	54
7	950287	23·48	998266	·19	952021	23·66	047979	53
8	951696	23·40	998255	·19	953441	23·60	046559	52
9	953100	23·32	998243	·19	954856	23·51	045144	51
10	954499	23·25	998232	·19	956267	23·44	043733	50
11	8·955894	23·17	9·998220	·19	8·957674	23·37	11·042326	49
12	957284	23·10	998209	·19	959075	23·29	040925	48
13	958670	23·02	998197	·19	960473	23·23	039527	47
14	960052	22·95	998186	·19	961866	23·14	038134	46
15	961429	22·88	998174	·19	963255	23·07	036745	45
16	962801	22·80	998163	·19	964639	23·00	035361	44
17	964170	22·73	998151	·19	966019	22·93	033981	43
18	965534	22·66	998139	·20	967394	22·86	032606	42
19	966893	22·59	998128	·20	968766	22·79	031234	41
20	968249	22·52	998116	·20	970133	22·71	029867	40
21	8·969600	22·44	9·998104	·20	8·071496	22·65	11·028504	39
22	970947	22·38	998092	·20	972855	22·57	027145	38
23	972289	22·31	998080	·20	974209	22·51	025791	37
24	973623	22·24	998068	·20	975560	22·44	024440	36
25	974962	22·17	998056	·20	976906	22·37	023094	35
26	976293	22·10	998044	·20	978248	22·30	021752	34
27	977619	22·03	998032	·20	979586	22·23	020414	33
28	978941	21·97	998020	·20	980921	22·17	019079	32
29	980259	21·90	998008	·20	982251	22·10	017749	31
30	981573	21·83	997996	·20	983577	22·04	016423	30
31	8·982883	21·77	9·997985	·20	8·984899	21·97	11·015101	29
32	984189	21·70	997972	·20	986217	21·91	013783	28
33	985491	21·63	997959	·20	987532	21·84	012468	27
34	986789	21·57	997947	·20	988842	21·78	011158	26
35	988083	21·50	997935	·21	990149	21·71	009851	25
36	989374	21·44	997922	·21	991451	21·65	008549	24
37	990660	21·38	997910	·21	992750	21·58	007250	23
38	991943	21·31	997897	·21	994045	21·52	005955	22
39	993222	21·25	997885	·21	995337	21·46	004663	21
40	994497	21·19	997872	·21	996624	21·40	003376	20
41	8·995768	21·12	9·997860	·21	8·997908	21·34	11·002092	19
42	997036	21·06	997847	·21	999188	21·27	000812	18
43	998299	21·00	997835	·21	9·000465	21·21	10·999535	17
44	999560	20·94	997822	·21	001738	21·15	998262	16
45	9·000816	20·87	997809	·21	003007	21·09	996993	15
46	002069	20·82	997797	·21	004272	21·03	995728	14
47	003318	20·76	997784	·21	005534	20·97	994466	13
48	004563	20·70	997771	·21	006792	20·91	993208	12
49	005805	20·64	997758	·21	008047	20·85	991953	11
50	007044	20·58	997745	·21	009298	20·80	990702	10
51	9·008278	20·52	9·997732	·21	9·010546	20·74	10·989454	9
52	009510	20·46	997719	·21	011790	20·68	988210	8
53	010737	20·40	997706	·21	013031	20·62	986969	7
54	011962	20·34	997693	·22	014268	20·56	985732	6
55	013182	20·29	997680	·22	015502	20·51	984498	5
56	014400	20·23	997667	·22	016732	20·45	983268	4
57	015613	20·17	997654	·22	017959	20·40	982041	3
58	016824	20·12	997641	·22	019183	20·33	980817	2
59	018031	20·06	997628	·22	020403	20·28	979597	1
60	019235	20·00	997614	·22	021620	20·23	978380	0
	Cosine	D.	Sine	84°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·019235	20·00	9·997614	·22	9·021620	20·23	10·978380	60
1	020435	19·95	997601	·22	022834	20·17	977166	59
2	021632	19·89	997588	·22	024044	20·11	975956	58
3	022825	19·84	997574	·22	025251	20·06	974749	57
4	024016	19·78	997561	·22	026455	20·00	973545	56
5	025203	19·73	997547	·22	027655	19·95	972345	55
6	026386	19·67	997534	·23	028852	19·90	971148	54
7	027567	19·62	997520	·23	030046	19·85	969954	53
8	028744	19·57	997507	·23	031237	19·79	968763	52
9	029918	19·51	997493	·23	032425	19·74	967575	51
10	031089	19·47	997480	·23	033609	19·69	966391	50
11	9·032257	19·41	9·997466	·23	9·034791	19·64	10·965209	49
12	033421	19·36	997452	·23	035969	19·58	964031	48
13	034582	19·30	997439	·23	037144	19·53	962856	47
14	035741	19·25	997425	·23	038316	19·48	961684	46
15	036896	19·20	997411	·23	039485	19·43	960515	45
16	038048	19·15	997397	·23	040651	19·38	959349	44
17	039197	19·10	997383	·23	041813	19·33	958187	43
18	040342	19·05	997369	·23	042973	19·28	957027	42
19	041485	18·99	997355	·23	044130	19·23	955870	41
20	042625	18·94	997341	·23	045284	19·18	954716	40
21	9·043762	18·89	9·997327	·24	9·046434	19·13	10·953566	39
22	044895	18·84	997313	·24	047582	19·08	952418	38
23	046026	18·79	697299	·24	048727	19·03	951273	37
24	047154	18·75	997285	·24	049869	18·98	950131	36
25	048279	18·70	997271	·24	051008	18·93	948992	35
26	049400	18·65	997257	·24	052144	18·89	947856	34
27	050519	18·60	997242	·24	053277	18·84	946723	33
28	051635	18·55	997228	·24	054407	18·79	945593	32
29	052749	18·50	997214	·24	055535	18·74	944465	31
30	053859	18·45	997199	·24	056659	18·70	943341	30
31	9·054966	18·41	9·997185	·24	9·057781	18·65	10·942219	29
32	056071	18·36	997170	·24	058900	18·69	941100	28
33	057172	18·31	997156	·24	060016	18·55	939984	27
34	058271	18·27	997141	·24	061130	18·51	938870	26
35	059367	18·22	997127	·24	062240	18·46	937760	25
36	060460	18·17	997112	·24	063348	18·42	936652	24
37	061551	18·13	997098	·24	064453	18·37	935547	23
38	062639	18·08	997083	·25	065556	18·38	934444	22
39	063724	18·04	997068	·25	066655	18·28	933345	21
40	064806	17·99	997053	·25	067752	18·24	932248	20
41	9·065885	17·94	9·997039	·25	9·068846	18·19	10·931154	19
42	066962	17·90	997024	·25	069938	18·15	930062	18
43	068036	17·86	997009	·25	071027	18·10	928973	17
44	069107	17·81	996994	·25	072113	18·06	927887	16
45	070176	17·77	996979	25	073197	18·02	926803	15
46	071242	17·72	996964	·25	074278	17·97	925722	14
47	072306	17·68	996949	·25	075356	17·93	924644	13
48	073366	17·63	996934	·25	076432	17·89	923568	12
49	074424	17·59	996919	·25	077505	17·84	922495	11
50	075480	17·55	996904	·25	078576	17·80	921424	10
51	9·076533	17·50	9·996889	·25	9·079644	17·76	10·920356	9
52	077583	17·46	996874	·25	080710	17·72	919290	8
53	078631	17·42	996858	·25	081773	17·67	918227	7
54	079676	17·38	996843	·25	08[illegible]833	17·63	917167	6
55	080719	17·33	996828	·25	083891	17·59	916109	5
56	081759	17·29	996812	·26	084947	17·55	915053	4
57	082797	17·25	996797	·26	086000	17·51	914000	3
58	083832	17·21	996782	·26	087050	17·47	912950	2
59	084864	17·17	996766	·26	088098	17·43	911902	1
60	085894	17·13	996751	·26	089144	17·38	910856	0
	Cosine	D.	Sine	83°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·085894	17·13	9·996751	·26	9·089144	17·38	10·910856	60
1	086922	17·09	996735	·26	090187	17·34	909813	59
2	087947	17·04	996720	·26	091228	17·30	908772	58
3	088970	17·00	996704	·26	092266	17·27	907734	57
4	089990	16·96	996688	·26	093302	17·22	906698	56
5	091008	16·92	996673	·26	094336	17·19	905664	55
6	092024	16·88	996657	·26	095367	17·15	904633	54
7	093037	16·84	996641	·26	096395	17·11	903605	53
8	094047	16·80	996625	·26	097422	17·07	902578	52
9	095056	16·76	996610	·26	098446	17·03	901554	51
10	096062	16·73	996594	·26	099468	16·99	900532	50
11	9·097065	16·68	9·996578	·27	9·100487	16·95	10·899513	49
12	098066	16·65	996562	·27	101504	16·91	898496	48
13	099065	16·61	996546	·27	102519	16·87	897481	47
14	100062	16·57	996530	·27	103532	16·84	896468	46
15	101056	16·53	996514	·27	104542	16·80	895458	45
16	102048	16·49	996498	·27	105550	16·76	894450	44
17	103037	16·45	996482	·27	106556	16·72	893444	43
18	104025	16·41	996465	·27	107559	16·69	892441	42
19	105010	16·38	996449	·27	108560	16·65	891440	41
20	105992	16·34	996433	·27	109559	16·61	890441	40
21	9·106973	16·30	9·996417	·27	9·110556	16·58	10·889444	39
22	107951	16·27	996400	·27	111551	16·54	888449	38
23	108927	16·23	996384	·27	112543	16·50	887457	37
24	109901	16·19	996368	·27	113533	16·46	886467	36
25	110873	16·16	996351	·27	114521	16·43	885479	35
26	111842	16·12	996335	·27	115507	16·39	884493	34
27	112809	16·08	996318	·27	116491	16·36	883509	33
28	113774	16·05	996302	·28	117472	16·32	882528	32
29	114737	16·01	996285	·28	118452	16·29	881548	31
30	115698	15·97	996269	·28	119429	16·25	880571	30
31	9·116656	15·94	9·996252	·28	9·120404	16·22	10·879596	29
32	117613	15·90	996235	·28	121377	16·18	878623	28
33	118567	15·87	996219	·28	122348	16·15	877652	27
34	119519	15·83	996202	·28	123317	16·11	876683	26
35	120469	15·80	996185	·28	124284	16·07	875716	25
36	121417	15·76	996168	·28	125249	16·04	874751	24
37	122362	15·73	996151	·28	126211	16·01	873789	23
38	123306	15·69	996134	·28	127172	15·97	872828	22
39	124248	15·66	996117	·28	128130	15·94	871870	21
40	125187	15·62	996100	·28	129087	15·91	870913	20
41	9·126125	15·59	9·996083	·29	9·130041	15·87	10·869959	19
42	127060	15·56	996066	·29	130994	15·84	869006	18
43	127993	15·52	996049	·29	131944	15·81	868056	17
44	128925	15·49	996032	·29	132893	15·77	867107	16
45	129854	15·45	996015	·29	133839	15·74	866161	15
46	130781	15·42	995998	·29	134784	15·71	865216	14
47	131706	15·39	995980	·29	135726	15·67	864274	13
48	132630	15·35	995963	·29	136667	15·64	863333	12
49	133551	15·32	995946	·29	137605	15·61	862395	11
50	134470	15·29	995928	·29	138542	15·58	861458	10
51	9·135387	15·25	9·995911	·29	9·139476	15·55	10·860524	9
52	136303	15·22	995894	·29	140409	15·51	859591	8
53	137216	15·19	995876	·29	141340	15·48	858660	7
54	138128	15·16	995859	·29	142269	15·45	857731	6
55	139037	15·12	995841	·29	143196	15·42	856804	5
56	139944	15·09	995823	·29	144121	15·39	855879	4
57	140850	15·06	995806	·29	145044	15·35	854956	3
58	141754	15·03	995788	·29	145966	15·32	854034	2
59	142655	15·00	995771	·29	146885	15·29	853115	1
60	143555	14·96	995753	·29	147803	15·26	852197	0
	Cosine	D.	Sine	82°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.143555	14.96	9.995753	.30	9.147803	15.26	10.852197	60
1	144453	14.93	995735	.30	148718	15.23	851282	59
2	145349	14.90	995717	.30	149632	15.20	850368	58
3	146243	14.87	995699	.30	150544	15.17	849456	57
4	147136	14.84	995681	.30	151454	15.14	848546	56
5	148026	14.81	995664	.30	152363	15.11	847637	55
6	148915	14.78	995646	.30	153269	15.08	846731	54
7	149802	14.75	995628	.30	154174	15.05	845826	53
8	150686	14.72	995610	.30	155077	15.02	844923	52
9	151569	14.69	995591	.30	155978	14.99	844022	51
10	152451	14.66	995573	.30	156877	14.96	843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	154208	14.60	995537	.30	158671	14.90	841329	48
13	155083	14.57	995519	.30	159565	14.87	840435	47
14	155957	14.54	995501	.31	160457	14.84	839543	46
15	156830	14.51	995482	.31	161347	14.81	838653	45
16	157700	14.48	995464	.31	162236	14.79	837764	44
17	158569	14.45	995446	.31	163123	14.76	836877	43
18	159435	14.42	995427	.31	164008	14.73	835992	42
19	160301	14.39	995409	.31	164892	14.70	835108	41
20	161164	14.36	995390	.31	165774	14.67	834226	40
21	9.162025	14.33	9.995372	.31	9.166654	14.64	10.833346	39
22	162885	14.30	995353	.31	167532	14.61	832468	38
23	163743	14.27	995334	.31	168409	14.58	831591	37
24	164600	14.24	995316	.31	169284	14.55	830716	36
25	165454	14.22	995297	.31	170157	14.53	829843	35
26	166307	14.19	995278	.31	171029	14.50	828971	34
27	167159	14.16	995260	.31	171899	14.47	828101	33
28	168008	14.13	995241	.32	172767	14.44	827233	32
29	168856	14.10	995222	.32	173634	14.42	826366	31
30	169702	14.07	995203	.32	174499	14.39	825501	30
31	9.170547	14.05	9.995184	.32	9.175362	14.36	10.824638	29
32	171389	14.02	995165	.32	176224	14.33	823776	28
33	172230	13.99	995146	.32	177084	14.31	822916	27
34	173070	13.96	995127	.32	177942	14.28	822058	26
35	173908	13.94	995108	.32	178799	14.25	821201	25
36	174744	13.91	995089	.32	179655	14.23	820345	24
37	175578	13.88	995070	.32	180508	14.20	819492	23
38	176411	13.86	995051	.32	181360	14.17	818640	22
39	177242	13.83	995032	.32	182211	14.15	817789	21
40	178072	13.80	995013	.32	183059	14.12	816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.09	10.816093	19
42	179726	13.74	994974	.32	184752	14.07	815248	18
43	180551	13.72	994955	.32	185597	14.04	814403	17
44	181374	13.69	994935	.32	186439	14.02	813561	16
45	182196	13.66	994916	.33	187280	13.99	812720	15
46	183016	13.64	994896	.33	188120	13.96	811880	14
47	183834	13.61	994877	.33	188958	13.93	811042	13
48	184651	13.59	994857	.33	189794	13.91	810206	12
49	185466	13.56	994838	.33	190629	13.89	809371	11
50	186280	13.53	994818	.33	191462	13.86	808538	10
51	9.187092	13.51	9.994798	.33	9.192294	13.84	10.807706	9
52	187903	13.48	994779	.33	193124	13.81	806876	8
53	188712	13.46	994759	.33	193953	13.79	806047	7
54	189519	13.43	994739	.33	194780	13.76	805220	6
55	190325	13.41	994719	.33	195606	13.74	804394	5
56	191130	13.38	994700	.33	196430	13.71	803570	4
57	191933	13.36	994680	.33	197253	13.69	802747	3
58	192734	13.33	994660	.33	198074	13.66	801926	2
59	193534	13.30	994640	.33	198894	13.64	801106	1
60	194332	13.28	994620	.33	199713	13.61	800287	0
	Cosine	D.	Sine	81°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·194332	13·28	9·994620	·33	9·199713	13·61	10·800287	60
1	195129	13·26	994600	·33	200529	13·59	799471	59
2	195925	13·23	994580	·33	201345	13·56	798655	58
3	196719	13·21	994560	·34	202159	13·54	797841	57
4	197511	13·18	994540	·34	202971	13·52	797029	56
5	198302	13·16	994519	·34	203782	13·49	796218	55
6	199091	13·13	994499	·34	204592	13·47	795408	54
7	199879	13·11	994479	·34	205400	13·45	794600	53
8	200666	13·08	994459	·34	206207	13·42	793793	52
9	201451	13·06	994438	·34	207013	13·40	792987	51
10	202234	13·04	994418	·34	207817	13·38	792183	50
11	9·203017	13·01	9·994397	·34	9·208619	13·35	10·791381	49
12	203797	12·99	994377	·34	209420	13·33	790580	48
13	204577	12·96	994357	·34	210220	13·31	789780	47
14	205354	12·94	994336	·34	211018	13·28	788982	46
15	206131	12·92	994316	·34	211815	13·26	788185	45
16	206906	12·89	994295	·34	212611	13·24	787389	44
17	207679	12·87	994274	·35	213405	13·21	786595	43
18	208452	12·85	994254	·35	214198	13·19	785802	42
19	209222	12·82	994233	·35	214989	13·17	785011	41
20	209992	12·80	994212	·35	215780	13·15	784220	40
21	9·210760	12·78	9·994191	·35	9·216568	13·12	10·783432	39
22	211526	12·75	994171	·35	217356	13·10	782644	38
23	212291	12·73	994150	·35	218142	13·08	781858	37
24	213055	12·71	994129	·35	218926	13·05	781074	36
25	213818	12·68	994108	·35	219710	13·03	780290	35
26	214579	12·66	994087	·35	220492	13·01	779508	34
27	215338	12·64	994066	·35	221272	12·99	778728	33
28	216097	12·61	994045	·35	222052	12·97	777948	32
29	216854	12·59	994024	·35	222830	12·94	777170	31
30	217609	12·57	994003	·35	223606	12·92	776394	30
31	9·218363	12·55	9·993981	·35	9·224382	12·90	10·775618	29
32	219116	12·53	993960	·35	225156	12·88	774844	28
33	219868	12·50	993939	·35	225929	12·86	774071	27
34	220618	12·48	993918	·35	226700	12·84	773300	26
35	221367	12·46	993896	·36	227471	12·81	772529	25
36	222115	12·44	993875	·36	228239	12·79	771761	24
37	222861	12·42	993854	·36	229007	12·77	770993	23
38	223606	12·39	993832	·36	229773	12·75	770227	22
39	224349	12·37	993811	·36	230539	12·73	769461	21
40	225092	12·35	993789	·36	231302	12·71	768698	20
41	9·225833	12·33	9·993768	·36	9·232065	12·69	10·767935	19
42	226573	12·31	993746	·36	232826	12·67	767174	18
43	227311	12·28	993725	·36	233586	12·65	766414	17
44	228048	12·26	993703	·36	234345	12·62	765655	16
45	228784	12·24	993681	·36	235103	12·60	764897	15
46	229518	12·22	993660	·36	235859	12·58	764141	14
47	230252	12·20	993638	·36	236614	12·56	763386	13
48	230984	12·18	993616	·36	237368	12·54	762632	12
49	231714	12·16	993594	·37	238120	12·52	761880	11
50	232444	12·14	993572	·37	238872	12·50	761128	10
51	9·233172	12·12	9·993550	·37	9·239622	12·48	10·760378	9
52	233899	12·09	993528	·37	240371	12·46	759629	8
53	234625	12·07	993506	·37	241118	12·44	758882	7
54	235349	12·05	993484	·37	241865	12·42	758135	6
55	236073	12·03	993462	·37	242610	12·40	757390	5
56	236795	12·01	993440	·37	243354	12·38	756646	4
57	237515	11·99	993418	·37	244097	12·36	755903	3
58	238235	11·97	993396	·37	244839	12·34	755161	2
59	238953	11·95	993374	·37	245579	12·32	754421	1
60	239670	11·93	993351	·37	246319	12·30	753681	0
	Cosine	D.	Sine	80°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·239670	11·93	9·993351	·37	9·246319	12·30	10·753681	60
1	240386	11·91	993329	·37	247057	12·28	752943	59
2	241101	11·89	993307	·37	247794	12·26	752206	58
3	241814	11·87	993285	·37	248530	12·24	751470	57
4	242526	11·85	993262	·37	249264	12·22	750736	56
5	243237	11·83	993240	·37	249998	12·20	750002	55
6	243947	11·81	993217	·38	250730	12·18	749270	54
7	244656	11·79	993195	·38	251461	12·17	748539	53
8	245363	11·77	993172	·38	252191	12·15	747809	52
9	246069	11·75	993149	·38	252920	12·13	747080	51
10	246775	11·73	993127	·38	253648	12·11	746352	50
11	9·247478	11·71	9·993104	·38	9·254374	12·09	10·745626	49
12	248181	11·69	993081	·38	255100	12·07	744900	48
13	248883	11·67	993059	·38	255824	12·05	744176	47
14	249583	11·65	993036	·38	256547	12·03	743453	46
15	250282	11·63	993013	·38	257269	12·01	742731	45
16	250980	11·61	992990	·38	257990	12·00	742010	44
17	251677	11·59	992967	·38	258710	11·98	741290	43
18	252373	11·58	992944	·38	259429	11·96	740571	42
19	253067	11·56	992921	·38	260146	11·94	739854	41
20	253761	11·54	992898	·38	260863	11·92	739137	40
21	9·254453	11·52	9·992875	·38	9·261578	11·90	10·738422	39
22	255144	11·50	992852	·38	262292	11·89	737708	38
23	255834	11·48	992829	·39	263005	11·87	736995	37
24	256523	11·46	992806	·39	263717	11·85	736283	36
25	257211	11·44	992783	·39	264428	11·83	735572	35
26	257898	11·42	992759	·39	265138	11·81	734862	34
27	258583	11·41	992736	·39	265847	11·79	734153	33
28	259268	11·39	992713	·39	266555	11·78	733445	32
29	259951	11·37	992690	·39	267261	11·76	732739	31
30	260633	11·35	992666	·39	267967	11·74	732033	30
31	9·261314	11·33	9·992643	·39	9·268671	11·72	10·731329	29
32	261994	11·31	992619	·39	269375	11·70	730625	28
33	262673	11·30	992596	·39	270077	11·69	729923	27
34	263351	11·28	992572	·39	270779	11·67	729221	26
35	264027	11·26	992549	·39	271479	11·65	728521	25
36	264703	11·24	992525	·39	272178	11·64	727822	24
37	265377	11·22	992501	·39	272876	11·62	727124	23
38	266051	11·20	992478	·40	273573	11·60	726427	22
39	266723	11·19	992454	·40	274269	11·58	725731	21
40	267395	11·17	992430	·40	274964	11·57	725036	20
41	9·268065	11·15	9·992406	·40	9·275658	11·55	10·724342	19
42	268734	11·13	992382	·40	276351	11·53	723649	18
43	269402	11·11	992359	·40	277043	11·51	722957	17
44	270069	11·10	992335	·40	277734	11·50	722266	16
45	270735	11·08	992311	·40	278424	11·48	721576	15
46	271400	11·06	992287	·40	279113	11·47	720887	14
47	272064	11·05	992263	·40	279801	11·45	720199	13
48	272726	11·03	992239	·40	280488	11·43	719512	12
49	273388	11·01	992214	·40	281174	11·41	718826	11
50	274049	10·99	992190	·40	281858	11·40	718142	10
51	9·274708	10·98	9·992166	·40	9·282542	11·38	10·717458	9
52	275367	10·96	992142	·40	283225	11·36	716775	8
53	276024	10·94	992117	·41	283907	11·35	716093	7
54	276681	10·92	992093	·41	284588	11·33	715412	6
55	277337	10·91	992069	·41	285268	11·31	714732	5
56	277991	10·89	992044	·41	285947	11·30	714053	4
57	278644	10·87	992020	·41	286624	11·28	713376	3
58	279297	10·86	991996	·41	287301	11·26	712699	2
59	279948	10·84	991971	·41	287977	11·25	712023	1
60	280599	10·82	991947	·41	288652	11·23	711348	0
	Cosine	D.	Sine	79°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·280599	10·82	9·991947	·41	9·288652	11·23	10·711348	60
1	281248	10·81	991922	·41	289326	11·22	710674	59
2	281897	10·79	991897	·41	289999	11·20	710001	58
3	282544	10·77	991873	·41	290671	11·18	709329	57
4	283190	10·76	991848	·41	291342	11·17	708658	56
5	283836	10·74	991823	·41	292013	11·15	707987	55
6	284480	10·72	991799	·41	292682	11·14	707318	54
7	285124	10·71	991774	·42	293350	11·12	706650	53
8	285766	10·69	991749	·42	294017	11·11	705983	52
9	286408	10·67	991724	·42	294684	11·09	705316	51
10	287048	10·66	991699	·42	295349	11·07	704651	50
11	9·287687	10·64	9·991674	·42	9·296013	11·06	10·703987	49
12	288326	10·63	991649	·42	296677	11·04	703323	48
13	288964	10·61	991624	·42	297339	11·03	702661	47
14	289600	10·59	991599	·42	298001	11·01	701999	46
15	290236	10·58	991574	·42	298662	11·00	701338	45
16	290870	10·56	991549	·42	299322	10·98	700678	44
17	291504	10·54	991524	·42	299980	10·96	700020	43
18	292137	10·53	991498	·42	300638	10·95	699362	42
19	292768	10·51	991473	·42	301295	10·93	698705	41
20	293399	10·50	991448	·42	301951	10·92	698049	40
21	9·294029	10·48	9·991422	·42	9·302607	10·90	10·697393	39
22	294658	10·46	991397	·42	303261	10·89	696739	38
23	295286	10·45	991372	·43	303914	10·87	696086	37
24	295913	10·43	991346	·43	304567	10·86	695433	36
25	296539	10·42	991321	·43	305218	10·84	694782	35
26	297164	10·40	991295	·43	305869	10·83	694131	34
27	297788	10·39	991270	·43	306519	10·81	693481	33
28	298412	10·37	991244	·43	307168	10·80	692832	32
29	299034	10·36	991218	·43	307815	10·78	692185	31
30	299655	10·34	991193	·43	308463	10·77	691537	30
31	9·300276	10·32	9·991167	·43	9·309109	10·75	10·690891	29
32	300895	10·31	991141	·43	309754	10·74	690246	28
33	301514	10·29	991115	·43	310398	10·73	689602	27
34	302132	10·28	991090	·43	311042	10·71	688958	26
35	302748	10·26	991064	·43	311685	10·70	688315	25
36	303364	10·25	991038	·43	312327	10·68	687673	24
37	303979	10·23	991012	·43	312967	10·67	687033	23
38	304593	10·22	990986	·43	313608	10·65	686392	22
39	305207	10·20	990960	·43	314247	10·64	685753	21
40	305819	10·19	990934	·44	314885	10·62	685115	20
41	9·306430	10·17	9·990908	·44	9·315523	10·61	10·684477	19
42	307041	10·16	990882	·44	316159	10·60	683841	18
43	307650	10·14	990855	·44	316795	10·58	683205	17
44	308259	10·13	990829	·44	317430	10·57	682570	16
45	308867	10·11	990803	·44	318064	10·55	681936	15
46	309474	10·10	990777	·44	318697	10·54	681303	14
47	310080	10·08	990750	·44	319329	10·53	680671	13
48	310685	10·07	990724	·44	319961	10·51	680039	12
49	311289	10·05	990697	·44	320592	10·50	679408	11
50	311893	10·04	990671	·44	321222	10·48	678778	10
51	9·312495	10·03	9·990644	·44	9·321851	10·47	10·678149	9
52	313097	10·01	990618	·44	322479	10·45	677521	8
53	313698	10·00	990591	·44	323106	10·44	676894	7
54	314297	9·98	990565	·44	323733	10·43	676267	6
55	314897	9·97	990538	·44	324358	10·41	675642	5
56	315495	9·96	990511	·45	324983	10·40	675017	4
57	316092	9·94	990485	·45	325607	10·39	674393	3
58	316689	9·93	990458	·45	326231	10·37	673769	2
59	317284	9·91	990431	·45	326853	10·36	673147	1
60	317879	9·90	990404	·45	327475	10·35	672525	0
	Cosine	D.	Sine	78°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·317879	9·90	9·990404	·45	9·327474	10·35	10·672526	60
1	318473	9·88	990378	·45	328095	10·33	671905	59
2	319066	9·87	990351	·45	328715	10·32	671285	58
3	319658	9·86	990324	·45	329334	10·30	670666	57
4	320249	9·84	990207	·45	329953	10·29	670047	56
5	320840	9·83	990270	·45	330570	10 28	669430	55
6	321430	9·82	990243	·45	331187	10·26	668813	54
7	322019	9·80	990215	·45	331803	10·25	668197	53
8	322607	9·79	990188	·45	332418	10·24	667582	52
9	323194	9·77	990161	·45	333033	10·23	666967	51
10	323780	9·76	990134	·45	333646	10·21	666354	50
11	9·324366	9·75	9·990107	·46	9·334259	10·20	10·665741	49
12	324950	9·73	990079	·46	334871	10·19	665129	48
13	325534	9·72	990052	·46	335482	10·17	664518	47
14	326117	9·70	990025	·46	336093	10·16	663907	46
15	326700	9·69	989997	·46	336702	10·15	663298	45
16	327281	9·68	989970	·46	337311	10·13	662689	44
17	327862	9·66	989942	·46	337919	10·12	662081	43
18	328442	9·65	989915	·46	338527	10·11	661473	42
19	329021	9·64	989887	·46	339133	10·10	660867	41
20	329599	9·62	989860	·46	339739	10·08	660261	40
21	9·330176	9·61	9·989832	·46	9·340344	10·07	10·659656	39
22	330753	9·60	989804	·46	340948	10·06	659052	38
23	331329	9·58	989777	·46	341552	10·04	658448	37
24	331903	9·57	989749	·47	342155	10·03	657845	36
25	332478	9·56	989721	·47	342757	10·02	657243	35
26	333051	9·54	989693	·47	343358	10·00	656642	34
27	333624	9·53	989665	·47	343958	9·99	656042	33
28	334195	9·52	989637	·47	344558	9·98	655442	32
29	334766	9·50	989609	·47	345157	9·97	654843	31
30	335337	9·49	989582	·47	345755	9·96	654245	30
31	9·335906	9·48	9·989553	·47	9·346353	9·94	10·653647	29
32	336475	9·46	989525	·47	346949	9·93	653051	28
33	337043	9·45	989497	·47	347545	9·92	652455	27
34	337610	9·44	989469	·47	348141	9·91	651859	26
35	338176	9·43	989441	·47	348735	9·90	651265	25
36	338742	9·41	989413	·47	349329	9·88	650671	24
37	339306	9·40	989384	·47	349922	9·87	650078	23
38	339871	9·39	989356	·47	350514	9·86	649486	22
39	340434	9·37	989328	·47	351106	9·85	648894	21
40	340996	9·36	989300	·47	351697	9·83	648303	20
41	9·341558	9·35	9·989271	·47	9·352287	9·82	10·647713	19
42	342119	9·34	989243	·47	352876	9·81	647124	18
43	342679	9·32	989214	·47	353465	9·80	646535	17
44	343239	9·31	989186	·47	354053	9·79	645947	16
45	343797	9·30	989157	·47	354640	9·77	645360	15
46	344355	9·29	989128	·48	355227	9·76	644773	14
47	344912	9·27	989100	·48	355813	9·75	644187	13
48	345469	9·26	989071	·48	356398	9·74	643602	12
49	346024	9·25	989042	·48	356982	9·73	643018	11
50	346579	9·24	989014	·48	357566	9·71	642434	10
51	9·347134	9·22	9·988985	·48	9·358149	9·70	10·641851	9
52	347687	9·21	988956	·48	358731	9·69	641269	8
53	348240	9·20	988927	·48	359313	9·68	640687	7
54	348792	9·19	988898	·48	359893	9·67	640107	6
55	349343	9·17	988869	·48	360474	9·66	639526	5
56	349893	9·16	988840	·48	361053	9·65	638947	4
57	350443	9·15	988811	·49	361632	9·63	638368	3
58	350992	9·14	988782	·49	362210	9·62	637790	2
59	351540	9·13	988753	·49	362787	9·61	637213	1
60	352088	9·11	988724	·49	363364	9·60	636636	0
	Cosine	D.	Sine	77°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	352635	9.10	988695	.49	363940	9.59	636060	59
2	353181	9.09	988666	.49	364515	9.58	635485	58
3	353726	9.08	988636	.49	365090	9.57	634910	57
4	354271	9.07	988607	.49	365664	9.55	634336	56
5	354815	9.05	988578	.49	366237	9.54	633763	55
6	355358	9.04	988548	.49	366810	9.53	633190	54
7	355901	9.03	988519	.49	367382	9.52	632618	53
8	356443	9.02	988489	.49	367953	9.51	632047	52
9	356984	9.01	988460	.49	368524	9.50	631476	51
10	357524	8.99	988430	.49	369094	9.49	630906	50
11	9.358064	8.98	9.988401	.49	9.369663	9.48	10.630337	49
12	358603	8.97	988371	.49	370232	9.46	629768	48
13	359141	8.96	988342	.49	370799	9.45	629201	47
14	359678	8.95	988312	.50	371367	9.44	628633	46
15	360215	8.93	988282	.50	371933	9.43	628067	45
16	360752	8.92	988252	.50	372499	9.42	627501	44
17	361287	8.91	988223	.50	373064	9.41	626936	43
18	361822	8.90	988193	.50	373629	9.40	626371	42
19	362356	8.89	988163	.50	374193	9.39	625807	41
20	362889	8.88	988133	.50	374756	9.38	625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	363954	8.85	988073	.50	375881	9.35	624119	38
23	364485	8.84	988043	.50	376442	9.34	623558	37
24	365016	8.83	988013	.50	377003	9.33	622997	36
25	365546	8.82	987983	.50	377563	9.32	622437	35
26	366075	8.81	987953	.50	378122	9.31	621878	34
27	366604	8.80	987922	.50	378681	9.30	621319	33
28	367131	8.79	987892	.50	379239	9.29	620761	32
29	367659	8.77	987862	.50	379797	9.28	620203	31
30	368185	8.76	987832	.51	380354	9.27	619646	30
31	9.368711	8.75	9.987801	.51	9.380910	9.26	10.619090	29
32	369236	8.74	987771	.51	381466	9.25	618534	28
33	369761	8.73	987740	.51	382020	9.24	617980	27
34	370285	8.72	987710	.51	382575	9.23	617425	26
35	370808	8.71	987679	.51	383129	9.22	616871	25
36	371330	8.70	987649	.51	383682	9.21	616318	24
37	371852	8.69	987618	.51	384234	9.20	615766	23
38	372373	8.67	987588	.51	384786	9.19	615214	22
39	372894	8.66	987557	.51	385337	9.18	614663	21
40	373414	8.65	987526	.51	385888	9.17	614112	20
41	9.373933	8.64	9.987496	.51	9.386438	9.15	10.613562	19
42	374452	8.63	987465	.51	386987	9.14	613013	18
43	374970	8.62	987434	.51	387536	9.13	612464	17
44	375487	8.61	987403	.52	388084	9.12	611916	16
45	376003	8.60	987372	.52	388631	9.11	611369	15
46	376519	8.59	987341	.52	389178	9.10	610822	14
47	377035	8.58	987310	.52	389724	9.09	610276	13
48	377549	8.57	987279	.52	390270	9.08	609730	12
49	378063	8.56	987248	.52	390815	9.07	609185	11
50	378577	8.54	987217	.52	391360	9.06	608640	10
51	9.379089	8.53	9.987186	.52	9.391903	9.05	10.608097	9
52	379601	8.52	987155	.52	392447	9.04	607553	8
53	380113	8.51	987124	.52	392989	9.03	607011	7
54	380624	8.50	987092	.52	393531	9.02	606469	6
55	381134	8.49	987061	.52	394073	9.01	605927	5
56	381643	8.48	987030	.52	394614	9.00	605386	4
57	382152	8.47	986998	.52	395154	8.99	604846	3
58	382661	8.46	986967	.52	395694	8.98	604306	2
59	383168	8.45	986936	.52	396233	8.97	603767	1
60	383675	8.44	986904	.52	396771	8.96	603229	0
	Cosine	D.	Sine	76°	Cotang.	D.	Tang	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.383675	8.44	9.986904	.52	9.396771	8.96	10.603229	60
1	384182	8.43	986873	.53	397309	8.96	602691	59
2	384687	8.42	986841	.53	397846	8.95	602154	58
3	385192	8.41	986809	.53	398383	8.94	601617	57
4	385697	8.40	986778	.53	398919	8.93	601081	56
5	386201	8.39	986746	.53	399455	8.92	600545	55
6	386704	8.38	986714	.53	399990	8.91	600010	54
7	387207	8.37	986683	.53	400524	8.90	599476	53
8	387709	8.36	986651	.53	401058	8.89	598942	52
9	388210	8.35	986619	.53	401591	8.88	598409	51
10	388711	8.34	986587	.53	402124	8.87	597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.86	10.597344	49
12	389711	8.32	986523	.53	403187	8.85	596813	48
13	390210	8.31	986491	.53	403718	8.84	596282	47
14	390708	8.30	986459	.53	404249	8.83	595751	46
15	391206	8.28	86427	.53	404778	8.82	595222	45
16	391703	8.27	986395	.53	405308	8.81	594692	44
17	392199	8.26	986363	.54	405836	8.80	594164	43
18	392695	8.25	986331	.54	406364	8.79	593636	42
19	393191	8.24	986299	.54	406892	8.78	593108	41
20	393685	8.23	986266	.54	407419	8.77	592581	40
21	9.394179	8.22	9.986234	.54	9.407945	8.76	10.592055	39
22	394673	8.21	986202	.54	408471	8.75	591529	38
23	395166	8.20	986169	.54	408997	8.74	591003	37
24	395658	8.19	986137	.54	409521	8.74	590479	36
25	396150	8.18	986104	.54	410045	8.73	589955	35
26	396641	8.17	986072	.54	410569	8.72	589431	34
27	397132	8.17	986039	.54	411092	8.71	588908	33
28	397621	8.16	986007	.54	411615	8.70	588385	32
29	398111	8.15	985974	.54	412137	8.69	587863	31
30	398600	8.14	985942	.54	412658	8.68	587342	30
31	9.399088	8.13	9.985909	.55	9.413179	8.67	10.586821	29
32	399575	8.12	985876	.55	413699	8.66	586301	28
33	400062	8.11	985843	.55	414219	8.65	585781	27
34	400549	8.10	985811	.55	414738	8.64	585262	26
35	401035	8.09	985778	.55	415257	8.64	584743	25
36	401520	8.08	985745	.55	415775	8.63	584225	24
37	402005	8.07	985712	.55	416293	8.62	583707	23
38	402489	8.06	985679	.55	416810	8.61	583190	22
39	402972	8.05	985646	.55	417326	8.60	582674	21
40	403455	8.04	985613	.55	417842	8.59	582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	404420	8.02	985547	.55	418873	8.57	581127	18
43	404901	8.01	985514	.55	419387	8.56	580613	17
44	405382	8.00	985480	.55	419901	8.55	580099	16
45	405862	7.99	985447	.55	420415	8.55	579585	15
46	406341	7.98	985414	.56	420927	8.54	579073	14
47	406820	7.97	985380	.56	421440	8.53	578560	13
48	407299	7.96	985347	.56	421952	8.52	578048	12
49	407777	7.95	985314	.56	422463	8.51	577537	11
50	408254	7.94	985280	.56	422974	8.50	577026	10
51	9.408731	7.94	9.985247	.56	9.423484	8.49	10.576516	9
52	409207	7.93	985213	.56	423993	8.48	576007	8
53	409682	7.92	985180	.56	424503	8.48	575497	7
54	410157	7.91	985146	.56	425011	8.47	574989	6
55	410632	7.90	985113	.56	425519	8.46	574481	5
56	411106	7.89	985079	.56	426027	8.45	573973	4
57	411579	7.88	985045	.56	426534	8.44	573466	3
58	412052	7.87	985011	.56	427041	8.43	572959	2
59	412524	7.86	984978	.56	427547	8.43	572453	1
60	412996	7.85	984944	56	428052	8.42	571948	0
	Cosine	D.	Sine	75°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9·412996	7·85	9·984944	·57	9·428052	8·42	10·571948	60
1	413467	7·84	984910	·57	428557	8·41	571443	59
2	413938	7·83	984876	·57	429062	8·40	570938	58
3	414408	7·83	984842	·57	429566	8·39	570434	57
4	414878	7·82	984808	·57	430070	8·38	569930	56
5	415347	7·81	984774	·57	430573	8·38	569427	55
6	415815	7·80	984740	·57	431075	8·37	568925	54
7	416283	7·79	984706	·57	431577	8·36	568423	53
8	416751	7·78	984672	·57	432079	8·35	567921	52
9	417217	7·77	984637	·57	432580	8·34	567420	51
10	417684	7·76	984603	·57	433080	8·33	566920	50
11	9·418150	7·75	9·984569	·57	9·433580	8·32	10·566420	49
12	418615	7·74	984535	·57	434080	8·32	565920	48
13	419079	7·73	984500	·57	434579	8·31	565421	47
14	419544	7·73	984466	·57	435078	8·30	564922	46
15	420007	7·72	984432	·58	435576	8·29	564424	45
16	420470	7·71	984397	·58	436073	8·28	563927	44
17	420933	7·70	984363	·58	436570	8·28	563430	43
18	421395	7·69	984328	·58	437067	8·27	562933	42
19	421857	7·68	984294	·58	437563	8·26	562437	41
20	422318	7·67	984259	·58	438059	8·25	561941	40
21	9·422778	7·67	9·984224	·58	9·438554	8·24	10·561446	39
22	423238	7·66	984190	·58	439048	8·23	560952	38
23	423697	7·65	984155	·58	439543	8·23	560457	37
24	424156	7·64	984120	·58	440036	8·22	559964	36
25	424615	7·63	984085	·58	440529	8·21	559471	35
26	425073	7·62	984050	·58	441022	8·20	558978	34
27	425530	7·61	984015	·58	441514	8·19	558486	33
28	425987	7·60	983981	·58	442006	8·19	557994	32
29	426443	7·60	983946	·58	442497	8·18	557503	31
30	426899	7·59	983911	·58	442988	8·17	557012	30
31	9·427354	7·58	9·983875	·58	9·443479	8·16	10·556521	29
32	427809	7·57	983840	·59	443968	8·16	556032	28
33	428263	7·56	983805	·59	444458	8·15	555542	27
34	428717	7·55	983770	·59	444947	8·14	555053	26
35	429170	7·54	983735	·59	445435	8·13	554565	25
36	429623	7·53	983700	·59	445923	8·12	554077	24
37	430075	7·52	983664	·59	446411	8·12	553589	23
38	430527	7·52	983629	·59	446898	8·11	553102	22
39	430978	7·51	983594	·59	447384	8·10	552616	21
40	431429	7·50	983558	·59	447870	8·09	552130	20
41	9·431879	7·49	9·983523	·59	9·448356	8·09	10·551644	19
42	432329	7·49	983487	·59	448841	8·08	551159	18
43	432778	7·48	983452	·59	449326	8·07	550674	17
44	433226	7·47	983416	·59	449810	8·06	550190	16
45	433675	7·46	983381	·59	450294	8·06	549706	15
46	434122	7·45	983345	·59	450777	8·05	549223	14
47	434569	7·44	983309	·59	451260	8·04	548740	13
48	435016	7·44	983273	·60	451743	8·03	548257	12
49	435462	7·43	983238	·60	452225	8·02	547775	11
50	435908	7·42	983202	·60	452706	8·02	547294	10
51	9·436353	7·41	9·983166	·60	9·453187	8·01	10·546813	9
52	436798	7·40	983130	·60	453668	8·00	546332	8
53	437242	7·40	983094	·60	454148	7·99	545852	7
54	437686	7·39	983058	·50	454628	7·99	545372	6
55	438129	7·38	983022	60	455107	7·98	544893	5
56	438572	7·37	982986	60	455586	7·97	544414	4
57	439014	7·36	982950	·60	456064	7·96	543936	3
58	439456	7·36	982914	·60	456542	7·96	543458	2
59	439897	7·35	982878	·60	457019	7·95	542981	1
60	440338	7·34	982842	·60	457496	7·94	542504	0
	Cosine	D.	Sine	74°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·440338	7·34	9·982842	·60	9·457496	7·94	10·542504	60
1	440778	7·33	982805	·60	457973	7·93	542027	59
2	441218	7·32	982769	·61	458449	7·93	541551	58
3	441658	7·31	982733	·61	458925	7·92	541075	57
4	442096	7·31	982696	·61	459400	7·91	540600	56
5	442535	7·30	982660	·61	459875	7·90	540125	55
6	442973	7·29	982624	·61	460349	7·90	539651	54
7	443410	7·28	982587	·61	460823	7·89	539177	53
8	443847	7·27	982551	·61	461297	7·88	538703	52
9	444284	7·27	982514	·61	461770	7·88	538230	51
10	444720	7·26	982477	·61	462242	7·87	537758	50
11	9·445155	7·25	0·982441	·61	9·462714	7·86	10·537286	49
12	445590	7·24	982404	·61	463186	7·85	536814	48
13	446025	7·23	982367	·61	463658	7·85	536342	47
14	446459	7·23	982331	·61	464129	7·84	535871	46
15	446893	7·22	982294	·61	464599	7·83	535401	45
16	447326	7·21	982257	·61	465069	7·83	534931	44
17	447759	7·20	982220	·62	465539	7·82	534461	43
18	448191	7·20	982183	·62	466008	7·81	533992	42
19	448623	7·19	982146	·62	466476	7·80	533524	41
20	449054	7·18	982109	·62	466945	7·80	533055	40
21	9·449485	7·17	9·982072	·62	9·467413	7·79	10·532587	39
22	449915	7·16	982035	·62	467880	7·78	532120	38
23	450345	7·16	981998	·62	468347	7·78	531653	37
24	450775	7·15	981961	·62	468814	7·77	531186	36
25	451204	7·14	981924	·62	469280	7·76	530720	35
26	451632	7·13	981886	·62	469746	7·75	530254	34
27	452060	7·13	981849	·62	470211	7·75	529789	33
28	452488	7·12	981812	·62	470676	7·74	529324	32
29	452915	7·11	981774	·62	471141	7·73	528859	31
30	453342	7·10	981737	·62	471605	7·73	528395	30
31	9·453768	7·10	9·981699	·63	9·472068	7·72	10·527932	29
32	454194	7·09	981662	·63	472532	7·71	527468	28
33	454619	7·08	981625	·63	472995	7·71	527005	27
34	455044	7·07	981587	·63	473457	7·70	526543	26
35	455469	7·07	981549	·63	473919	7·69	526081	25
36	455893	7·06	981512	·63	474381	7·69	525619	24
37	456316	7·05	981474	·63	474842	7·68	525158	23
38	456739	7·04	981436	·63	475303	7·67	524697	22
39	457162	7·04	981399	·63	475763	7·67	524237	21
40	457584	7·03	981361	·63	476223	7·66	523777	20
41	9·458006	7·02	9·981323	·63	9·476683	7·65	10·523317	19
42	458427	7·01	981285	·63	477142	7·65	522858	18
43	458848	7·01	981247	·63	477601	7·64	522399	17
44	459268	7·00	981209	·63	478059	7·63	521941	16
45	459688	6·99	981171	·63	478517	7·63	521483	15
46	460108	6·98	981133	·64	478975	7·62	521025	14
47	460527	6·98	981095	·64	479432	7·61	520568	13
48	460946	6·97	981057	·64	479889	7·61	520111	12
49	461364	6·96	981019	·64	480345	7·60	519655	11
50	461782	6·95	980981	·64	480801	7·59	519199	10
51	9·462199	6·95	9·980942	·64	9·481257	7·59	10·518743	9
52	462616	6·94	980904	·64	481712	7·58	518288	8
53	463032	6·93	980866	·64	482167	7·57	517833	7
54	463448	6·93	980827	·64	482621	7·57	517379	6
55	463864	6·92	980789	·64	483075	7·56	516925	5
56	464279	6·91	980750	·64	483529	7·55	516471	4
57	464694	6·90	980712	·64	483982	7·55	516018	3
58	465108	6·90	980673	·64	484435	7·54	515565	2
59	465522	6·89	980635	·64	484887	7·53	515113	1
60	465935	6·88	980596	·64	485339	7·53	514661	0
	Cosine	D.	Sine	73°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·465935	6·88	9·980596	·64	9·485339	7·53	10·514661	60
1	466348	6·88	980558	·64	485791	7·52	514209	59
2	466761	6·87	980519	·65	486242	7·51	513758	58
3	467173	6·86	980480	·65	486693	7·51	513307	57
4	467585	6·85	980442	·65	487143	7·50	512857	56
5	467996	6·85	980403	·65	487593	7·49	512407	55
6	468407	6·84	980364	·65	488043	7·49	511957	54
7	468817	6·83	980325	·65	488492	7·48	511508	53
8	469227	6·83	980286	·65	488941	7·47	511059	52
9	469637	6·82	980247	·65	489390	7·47	510610	51
10	470046	6·81	980208	·65	489838	7·46	510162	50
11	9·470455	6·80	9·980169	·65	9·490286	7·46	10·509714	49
12	470863	6·80	980130	·65	490733	7·45	509267	48
13	471271	6·79	980091	·65	491180	7·44	508820	47
14	471679	6·78	980052	·65	491627	7·44	508373	46
15	472086	6·78	980012	·65	492073	7·43	507927	45
16	472492	6·77	979973	·65	492519	7·43	507481	44
17	472898	6·76	979934	·66	492965	7·42	507035	43
18	473304	6·76	979895	·66	493410	7·41	506590	42
19	473710	6·75	979855	·66	493854	7·40	506146	41
20	474115	6·74	979816	·66	494299	7·40	505701	40
21	9·474519	6·74	9·979776	·66	9·494743	7·40	10·505257	39
22	474923	6·73	979737	·66	495186	7·39	504814	38
23	475327	6·72	979697	·66	495630	7·38	504370	37
24	475730	6·72	979658	·66	496073	7·37	503927	36
25	476133	6·71	979618	·66	496515	7·37	503485	35
26	476536	6·70	979579	·66	496957	7·36	503043	34
27	476938	6·69	979539	·66	497399	7·36	502601	33
28	477340	6·69	979499	·66	497841	7·35	502159	32
29	477741	6·68	979459	·66	498282	7·34	501718	31
30	478142	6·67	979420	·66	498722	7·34	501278	30
31	9·478542	6·67	9·979380	·66	9·499163	7·33	10·500837	29
32	478942	6·66	979340	·66	499603	7·33	500397	28
33	479342	6·65	979300	·67	500042	7·32	499958	27
34	479741	6·65	979260	·67	500481	7·31	499519	26
35	480140	6·64	979220	·67	500920	7·31	499080	25
36	480539	6·63	979180	·67	501359	7·30	498641	24
37	480937	6·63	979140	·67	501797	7·30	498203	23
38	481334	6·62	979100	·67	502235	7·29	497765	22
39	481731	6·61	979059	·67	502672	7·28	497328	21
40	482128	6·61	979019	·67	503109	7·28	496891	20
41	9·482525	6·60	9·978979	·67	9·503546	7·27	10·496454	19
42	482921	6·59	978939	·67	503982	7·27	496018	18
43	483316	6·59	978898	·67	504418	7·26	495582	17
44	483712	6·58	978858	·67	504854	7·25	495146	16
45	484107	6·57	978817	·67	505289	7·25	494711	15
46	484501	6·57	978777	·67	505724	7·24	494276	14
47	484895	6·56	978736	·67	506159	7·24	493841	13
48	485289	6·55	978696	·68	506593	7·23	493407	12
49	485682	6·55	978655	·68	507027	7·22	492973	11
50	486075	6·54	978615	·68	507460	7·22	492540	10
51	9·486467	6·53	9·978574	·68	9·507893	7·21	10·492107	9
52	486860	6·53	978533	·68	508326	7·21	491674	8
53	487251	6·52	978493	·68	508759	7·20	491241	7
54	487643	6·51	978452	·68	509191	7·19	490809	6
55	488034	6·51	978411	·68	509622	7·19	490378	5
56	488424	6·50	978370	·68	510054	7·18	489946	4
57	488814	6·50	978329	·68	510485	7·18	489515	3
58	489204	6·49	978288	·68	510916	7·17	489084	2
59	489593	6·48	978247	·68	511346	7·16	488654	1
60	489982	6·48	978206	·68	511776	7·16	488224	0
	Cosine	D.	Sine	72°	Cotang.	D.	Tang.	M

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·489982	6·48	9·978206	·68	9·511776	7·16	10·488224	60
1	490371	6·48	978165	·68	512206	7·16	487794	59
2	490759	6·47	978124	·68	512635	7·15	487365	58
3	491147	6·46	978083	·69	513064	7·14	486936	57
4	491535	6·46	978042	·69	513493	7·14	486507	56
5	491922	6·45	978001	·69	513921	7·13	486079	55
6	492308	6·44	977959	·69	514349	7·13	485651	54
7	492695	6·44	977918	·69	514777	7·12	485223	53
8	493081	6·43	977877	·69	515204	7·12	484796	52
9	493466	6·42	977835	·69	515631	7·11	484369	51
10	493851	6·42	977794	·69	516057	7·10	483943	50
11	9·494236	6·41	9·977752	·69	9·516484	7·10	10·483516	49
12	494621	6·41	977711	·69	516910	7·09	483090	48
13	495005	6·40	977669	·69	517335	7·09	482665	47
14	495388	6·39	977628	·69	517761	7·08	482239	46
15	495772	6·39	977586	·69	518185	7·08	481815	45
16	496154	6·38	977544	·70	518610	7·07	481390	44
17	496537	6·37	977503	·70	519034	7·06	480966	43
18	496919	6·37	977461	·70	519458	7·06	480542	42
19	497301	6·36	977419	·70	519882	7·05	480118	41
20	497682	6·36	977377	·70	520305	7·05	479695	40
21	9·498064	6·35	9·977335	·70	9·520728	7·04	10·479272	39
22	498444	6·34	977293	·70	521151	7·03	478849	38
23	498825	6·34	977251	·70	521573	7·03	478427	37
24	499204	6·33	977209	·70	521995	7·03	478005	36
25	499584	6·32	977167	·70	522417	7·02	477583	35
26	499963	6·32	977125	·70	522838	7·02	477162	34
27	500342	6·31	977083	·70	523259	7·01	476741	33
28	500721	6·31	977041	·70	523680	7·01	476320	32
29	501099	6·30	976999	·70	524100	7·00	475900	31
30	501476	6·29	976957	·70	524520	6·99	475480	30
31	9·501854	6·29	9·976914	·70	9·524939	6·99	10·475061	29
32	502231	6·28	976872	·71	525359	6·98	474641	28
33	502607	6·28	976830	·71	525778	6·98	474222	27
34	502984	6·27	976787	·71	526197	6·97	473803	26
35	503360	6·26	976745	·71	526615	6·97	473385	25
36	503735	6·26	976702	·71	527033	6·96	472967	24
37	504110	6·25	976660	·71	527451	6·96	472549	23
38	504485	6·25	976617	·71	527868	6·95	472132	22
39	504860	6·24	976574	·71	528285	6·95	471715	21
40	505234	6·23	976532	·71	528702	6·94	471298	20
41	9·505608	6·23	9·976489	·71	9·529119	6·93	10·470881	19
42	505981	6·22	976446	·71	529535	6·93	470465	18
43	506354	6·22	976404	·71	529950	6·93	470050	17
44	506727	6·21	976361	·71	530366	6·92	469634	16
45	507099	6·20	676318	·71	530781	6·91	469219	15
46	507471	6·20	976275	·71	531196	6·91	468804	14
47	507843	6·19	976232	·72	531611	6·90	468389	13
48	508214	6·19	976189	·72	532025	6·90	467975	12
49	508585	6·18	976146	·72	532439	6·89	467561	11
50	508956	6·18	976103	·72	532853	6·89	467147	10
51	9·509326	6·17	9·976060	·72	9·533266	6·88	10·466734	9
52	509696	6·16	976017	·72	533679	6·88	466321	8
53	510065	6·16	975974	·72	534092	6·87	465908	7
54	510434	6·15	975930	·72	534504	6·87	465496	6
55	510803	6·15	975887	·72	534916	6·86	465084	5
56	511172	6·14	975844	·72	535328	6·86	464672	4
57	511540	6·13	975800	·72	535739	6·85	464261	3
58	511907	6·13	975757	·72	536150	6·85	463850	2
59	512275	6·12	975714	·72	536561	6·84	463439	1
60	512642	6·12	975670	·72	536972	6·84	463028	0
	Cosine	D.	Sine	D.	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·512642	6·12	9·975670	·73	9·536972	6·84	10·463028	60
1	513009	6·11	975627	·73	537382	6·83	462618	59
2	513375	6·11	975583	·73	537792	6·83	462208	58
3	513741	6·10	975539	·73	538202	6·82	461798	57
4	514107	6·09	975496	·73	538611	6·82	461389	56
5	514472	6·09	975452	·73	539020	6·81	460980	55
6	514837	6·08	975408	·73	539429	6·81	460571	54
7	515202	6·08	975365	·73	539837	6·80	460163	53
8	515566	6·07	975321	·73	540245	6·80	459755	52
9	515930	6·07	975277	·73	540653	6·79	459347	51
10	516294	6·06	975233	·73	541061	6·79	458939	50
11	9·516657	6·05	9·975189	·73	9·541468	6·78	10·458532	49
12	517020	6·05	975145	·73	541875	6·78	458125	48
13	517382	6·04	975101	·73	542281	6·77	457719	47
14	517745	6·04	975057	·73	542688	6·77	457312	46
15	518107	6·03	975013	·73	543094	6·76	456906	45
16	518468	6·03	974969	·74	543499	6·76	456501	44
17	518829	6·02	974925	·74	543905	6·75	456095	43
18	519190	6·01	974880	·74	544310	6·75	455690	42
19	519551	6·01	974836	·74	544715	6·74	455285	41
20	519911	6·00	974792	·74	545119	6·74	454881	40
21	9·520271	6·00	9·974748	·74	9·545524	6·73	10·454476	39
22	520631	5·99	974703	·74	545928	6·73	454072	38
23	520990	5·99	974659	·74	546331	6·72	453669	37
24	521349	5·98	974614	·74	546735	6·72	453265	36
25	521707	5·98	974570	·74	547138	6·71	452862	35
26	522066	5·97	974525	·74	547540	6·71	452460	34
27	522424	5·96	974481	·74	547943	6·70	452057	33
28	522781	5·96	974436	·74	548345	6·70	451655	32
29	523138	5·95	974391	·74	548747	6·69	451253	31
30	523495	5·95	974347	·75	549149	6·69	450851	30
31	9·523852	5·94	9·974302	·75	9·549550	6·68	10·450450	29
32	524208	5·94	974257	·75	549951	6·68	450049	28
33	524564	5·93	974212	·75	550352	6·67	449648	27
34	524920	5·93	974167	·75	550752	6·67	449248	26
35	525275	5·92	974122	·75	551152	6·66	448848	25
36	525630	5·91	974077	·75	551552	6·66	448448	24
37	525984	5·91	974032	·75	551952	6·65	448048	23
38	526339	5·90	973987	·75	552351	6·65	447649	22
39	526693	5·90	973942	·75	552750	6·65	447250	21
40	527046	5·89	973897	·75	553149	6·64	446851	20
41	9·527400	5·89	9·973852	·75	9·553548	6·64	10·446452	19
42	527753	5·88	973807	·75	553946	6·63	446054	18
43	528105	5·88	973761	·75	554344	6·63	445656	17
44	528458	5·87	973716	·76	554741	6·62	445259	16
45	528810	5·87	973671	·76	555139	6·62	444861	15
46	529161	5·86	973625	·76	555536	6·61	444464	14
47	529513	5·86	973580	·76	555933	6·61	444067	13
48	529864	5·85	973535	·76	556329	6·60	443671	12
49	530215	5·85	973489	·76	556725	6·60	443275	11
50	530565	5·84	973444	·76	557121	6·59	442879	10
51	9·530915	5·84	9·973398	·76	9·557517	6·59	10·442483	9
52	531265	5·83	973352	·76	557913	6·59	442087	8
53	531614	5·82	973307	·76	558308	6·58	441692	7
54	531963	5·82	973261	·76	558702	6·58	441298	6
55	532312	5·81	973215	·76	559097	6·57	440903	5
56	532661	5·81	973169	·76	559491	6·57	440509	4
57	533009	5·80	973124	·76	559885	6·56	440115	3
58	533357	5·80	973078	·76	560279	6·56	439721	2
59	533704	5·79	973032	·77	560673	6·55	439327	1
60	534052	5·78	972986	·77	561066	6·55	438934	0
	Cosine	D.	Sine	70°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·534052	5·78	9·972986	·77	9·561066	6·55	10·438934	60
1	534399	5·77	972940	·77	561459	6·54	438541	59
2	534745	5·77	972894	·77	561851	6·54	438149	58
3	535092	5·77	972848	·77	562244	6·53	437756	57
4	535438	5·76	972802	·77	562636	6·53	437364	56
5	535783	5·76	972755	·77	563028	6·53	436972	55
6	536129	5·75	972709	·77	563419	6·52	436581	54
7	536474	5·74	972663	·77	563811	6·52	436189	53
8	536818	5·74	972617	·77	564202	6·51	435798	52
9	537163	5·73	972570	·77	564592	6·51	435408	51
10	537507	5·73	972524	·77	564983	6·50	435017	50
11	9·537851	5·72	9·972478	·77	9·565373	6·50	10·434627	49
12	538194	5·72	972431	·78	565763	6·49	434237	48
13	538538	5·71	972385	·78	566153	6·49	433847	47
14	538880	5·71	972338	·78	566542	6·49	433458	46
15	539223	5·70	972291	·78	566932	6·48	433068	45
16	539565	5·70	972245	·78	567320	6·48	432680	44
17	539907	5·69	972198	·78	567709	6·47	432291	43
18	540249	5·69	972151	·78	568098	6·47	431902	42
19	540590	5·68	972105	·78	568486	6·46	431514	41
20	540931	5·68	972058	·78	568873	6·46	431127	40
21	9·541272	5·67	9·972011	·78	9·569261	6·45	10·430739	39
22	541613	5·67	971964	·78	569648	6·45	430352	38
23	541953	5·66	971917	·78	570035	6·45	429965	37
24	542293	5·66	971870	·78	570422	6·44	429578	36
25	542632	5·65	971823	·78	570809	6·44	429191	35
26	542971	5·65	971776	·78	571195	6·43	428805	34
27	543310	5·64	971729	·79	571581	6·43	428419	33
28	543649	5·64	971682	·79	571967	6·42	428033	32
29	543987	5·63	971635	·79	572352	6·42	427648	31
30	544325	5·63	971588	·79	572738	6·42	427262	30
31	9·544663	5·62	9·971540	·79	9·573123	6·41	10·426877	29
32	545000	5·62	971493	·79	573507	6·41	426493	28
33	545338	5·61	971446	·79	573892	6·40	426108	27
34	545674	5·61	971398	·79	574276	6·40	425724	26
35	546011	5·60	971351	·79	574660	6·39	425340	25
36	546347	5·60	971303	·79	575044	6·39	424956	24
37	546683	5·59	971256	·79	575427	6·39	424573	23
38	547019	5·59	971208	·79	575810	6·38	424190	22
39	547354	5·58	971161	·79	576193	6·38	423807	21
40	547689	5·58	971113	·79	576576	6·37	423424	20
41	9·548024	5·57	9·971066	·80	9·576958	6·37	10·423041	19
42	548359	5·57	971018	·80	577341	6·36	422659	18
43	548693	5·56	970970	·80	577723	6·36	422277	17
44	549027	5·56	970922	·80	578104	6·36	421896	16
45	549360	5·55	970874	·80	578486	6·35	421514	15
46	549693	5·55	970827	·80	578867	6·35	421133	14
47	550026	5·54	970779	·80	579248	6·34	420752	13
48	550359	5·54	970731	·80	579629	6·34	420371	12
49	550692	5·53	970683	·80	580009	6·34	419991	11
50	551024	5·53	970635	·80	580389	6·33	419611	10
51	9·551356	5·52	9·970586	·80	9·580769	6·33	10·419231	9
52	551687	5·52	970538	·80	581149	6·32	418851	8
53	552018	5·52	970490	·80	581528	6·32	418472	7
54	552349	5·51	970442	·80	581907	6·32	418093	6
55	552680	5·51	970394	·80	582286	6·31	417714	5
56	553010	5·50	970345	·81	582665	6·31	417335	4
57	553341	5·50	970297	·81	583043	6·30	416957	3
58	553670	5·49	970249	·81	583422	6·30	416578	2
59	554000	5·49	970200	·81	583800	6·29	416200	1
60	554329	5·48	970152	·81	584177	6·29	415823	0
	Cosine	D.	Sine	69°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·554329	5·48	9·970152	·81	9·584177	6·29	10·415823	60
1	554658	5·48	970103	·81	584555	6·29	415445	59
2	554987	5·47	970055	·81	584932	6·28	415068	58
3	555315	5·47	970006	·81	585309	6·28	414691	57
4	555643	5·46	969957	·81	585686	6·27	414314	56
5	555971	5·46	969909	·81	586062	6·27	413938	55
6	556299	5·45	969860	·81	586439	6·27	413561	54
7	556626	5·45	969811	·81	586815	6·26	413185	53
8	556953	5·44	969762	·81	587190	6·26	412810	52
9	557280	5·44	969714	·81	587566	6·25	412434	51
10	557606	5·43	969665	·81	587941	6·25	412059	50
11	9·557932	5·43	9·969616	·82	9·588316	6·25	10·411684	49
12	558258	5·43	969567	·82	588691	6·24	411309	48
13	558583	5·42	969518	·82	589066	6·24	410934	47
14	558909	5·42	969469	·82	589440	6·23	410560	46
15	559234	5·41	969420	·82	589814	6·23	410186	45
16	559558	5·41	969370	·82	590188	6·23	409812	44
17	559883	5·40	969321	·82	590562	6·22	409438	43
18	560207	5·40	969272	·82	590935	6·22	409065	42
19	560531	5·39	969223	·82	591308	6·22	408692	41
20	560855	5·39	969173	·82	591681	6·21	408319	40
21	9·561178	5·38	9·969124	·82	9·592054	6·21	10·407946	39
22	561501	5·38	969075	·82	592426	6·20	407574	38
23	561824	5·37	969025	·82	592798	6·20	407202	37
24	562146	5·37	968976	·82	593170	6·19	406829	36
25	562468	5·36	968926	·83	593542	6·19	406458	35
26	562790	5·36	968877	·83	593914	6·18	406086	34
27	563112	5·36	968827	·83	594285	6·18	405715	33
28	563433	5·35	968777	·83	594656	6·18	405344	32
29	563755	5·35	968728	·83	595027	6·17	404973	31
30	564075	5·34	968678	·83	595398	6·17	404602	30
31	9·564396	5·34	9·968628	·83	9·595768	6·17	10·404232	29
32	564716	5·33	968578	·83	596138	6·16	403862	28
33	565036	5·33	968528	·83	596508	6·16	403492	27
34	565356	5·32	968479	·83	596878	6·16	403122	26
35	565676	5·32	968429	·83	597247	6·15	402753	25
36	565995	5·31	968379	·83	597616	6·15	402384	24
37	566314	5·31	968329	·83	597985	6·15	402015	23
38	566632	5·31	963278	·83	598354	6·14	401646	22
39	566951	5·30	968228	·84	598722	6·14	401278	21
40	567269	5·30	968178	·84	599091	6·13	400909	20
41	9·567587	5·29	9·968128	·84	9·599459	6·13	10·400541	19
42	567904	5·29	968078	·84	599827	6·13	400173	18
43	568222	5·28	968027	·84	600194	6·12	399806	17
44	568539	5·28	967977	·84	600562	6·12	399438	16
45	568856	5·28	967927	·84	600929	6·11	399071	15
46	569172	5·27	967876	·84	601296	6·11	398704	14
47	569488	5·27	967826	·84	601662	6·11	398338	13
48	569804	5·26	967775	·84	602029	6·10	397971	12
49	570120	5·26	967725	·84	602395	6·10	397605	11
50	570435	5·25	967674	·84	602761	6·10	397239	10
51	9·570751	5·25	9·967624	·84	9·603127	6·09	10·396873	9
52	571066	5·24	967573	·84	603493	6·09	396507	8
53	571380	5·24	967522	·85	603858	6·09	396142	7
54	571695	5·23	967471	·85	604223	6·08	395777	6
55	572009	5·23	967421	·85	604588	6·08	395412	5
56	572323	5·23	967370	·85	604953	6·07	395047	4
57	572636	5·22	967319	·85	605317	6·07	394683	3
58	572950	5·22	967268	·85	605682	6·07	394318	2
59	573263	5·21	967217	·85	606046	6·06	393954	1
60	573575	5·21	967166	·85	606410	6·06	393590	0
	Cosine	D.	Sine	68°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·573575	5·21	9·967166	·85	9·606410	6·06	10·393590	60
1	573888	5·20	967115	·85	606773	6·06	393227	59
2	574200	5·20	967064	·85	607137	6·05	392863	58
3	574512	5·19	967013	·85	607500	6·05	392500	57
4	574824	5·19	966961	·85	607863	6·04	392137	56
5	575136	5·19	966910	·85	608225	6·04	391775	55
6	575447	5·18	966859	·85	608588	6·04	391412	54
7	575758	5·18	966808	·85	608950	6·03	391050	53
8	576069	5·17	966756	·86	609312	6·03	390688	52
9	576379	5·17	966705	·86	609674	6·03	390326	51
10	576689	5·16	966653	·86	610036	6·02	389964	50
11	9·576999	5·16	9·966602	·86	9·610397	6·02	10·389603	49
12	577309	5·16	966550	·86	610759	6·02	389241	48
13	577618	5·15	966499	·86	611120	6·01	388880	47
14	577927	5·15	966447	·86	611480	6·01	388520	46
15	578236	5·14	966395	·86	611841	6·01	388159	45
16	578545	5·14	966344	·86	612201	6·00	387799	44
17	578853	5·13	966292	·86	612561	6·00	387439	43
18	579162	5·13	966240	·86	612921	6·00	387079	42
19	579470	5·13	966188	·86	613281	5·99	386719	41
20	579777	5·12	966136	·86	613641	5·99	386359	40
21	9·580085	5·12	9·966085	·87	9·614000	5·98	10·386000	39
22	580392	·11	966033	·87	614359	5·98	385641	38
23	580699	5·11	965981	·87	614718	5·98	385282	37
24	581005	5·11	965928	·87	615077	5·97	384923	36
25	581312	5·10	965876	·87	615435	5·97	384565	35
26	581618	5·10	965824	·87	615793	5·97	384207	34
27	581924	5·09	965772	·87	616151	5·96	383849	33
28	582229	5·09	965720	·87	616509	5·96	383491	32
29	582535	5·09	965668	·87	616867	5·96	383133	31
30	582840	5·08	965615	·87	617224	5·95	382776	30
31	9·583145	5·08	9·965563	·87	9·617582	5·95	10·382418	29
32	583449	5·07	965511	·87	617939	5·95	382061	28
33	583754	5·07	965458	·87	618295	5·94	381705	27
34	584058	5·06	965406	·87	618652	5·94	381348	26
35	584361	5·06	965353	·88	619008	5·94	380992	25
36	584665	5·06	965301	·88	619364	5·93	380636	24
37	584968	5·05	965248	·88	619721	5·93	380279	23
38	585272	5·05	965195	·88	620076	5·93	379924	22
39	585574	5·04	965143	·88	620432	5·92	379568	21
40	585877	5·04	965090	·88	620787	5·92	379213	20
41	9·586179	5·03	9·965037	·88	9·621142	5·92	10·378858	19
42	586482	5·03	964984	·88	621497	5·91	378503	18
43	586783	5·03	964931	·88	621852	5·91	378148	17
44	587085	5·02	964879	·88	622207	5·90	377793	16
45	587386	5·02	964826	·88	622561	5·90	377439	15
46	587688	5·01	964773	·88	622915	5·90	377085	14
47	587989	5·01	964719	·88	623269	5·89	376731	13
48	588289	5·01	964666	·89	623623	5·89	376377	12
49	588590	5·00	964613	·89	623976	5·89	376024	11
50	588890	5·00	964560	·89	624330	5·88	375670	10
51	9·589190	4·99	9·964507	·89	9·624683	5·88	10·375317	9
52	589489	4·99	964454	·89	625036	5·88	374964	8
53	589789	4·99	964400	·89	625388	5·87	374612	7
54	590088	4·98	964347	·89	625741	5·87	374259	6
55	590387	4·98	964294	·89	626093	5·87	373907	5
56	590686	4·97	964240	·89	626445	5·86	373555	4
57	590984	4·97	964187	·89	626797	5·86	373203	3
58	591282	4·97	964133	·89	627149	5·86	372851	2
59	591580	4·96	964080	·89	627501	5·85	372499	1
60	591878	4·96	964026	·89	627852	5·85	372148	0
	Cosine	D.	Sine	67°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·591878	4·96	9·964026	·89	9·627852	5·85	10·372148	60
1	592176	4·95	963972	·89	628203	5·85	371797	59
2	592473	4·95	963919	·89	628554	5·85	371446	58
3	592770	4·95	963865	·90	628905	5·84	371095	57
4	593067	4·94	963811	·90	629255	5·84	370745	56
5	593363	4·94	963757	·90	629606	5·83	370394	55
6	593659	4·93	963704	·90	629956	5·83	370044	54
7	593955	4·93	963650	·90	630306	5·83	369694	53
8	594251	4·93	963596	·90	630656	5·83	369344	52
9	594547	4·92	963542	·90	631005	5·82	368995	51
10	594842	4·92	963488	·90	631355	5·82	368645	50
11	9·595137	4·91	9·963434	·90	9·631704	5·82	10·368296	49
12	595432	4·91	963379	·90	632053	5·81	367947	48
13	595727	4·91	963325	·90	632401	5·81	367599	47
14	596021	4·90	963271	·90	632750	5·81	367250	46
15	596315	4·90	963217	·90	633098	5·80	366902	45
16	596609	4·89	963163	·90	633447	5·80	366553	44
17	596903	4·89	963108	·91	633795	5·80	366205	43
18	597196	4·89	963054	·91	634143	5·79	365857	42
19	597490	4·88	962999	·91	634490	5·79	365510	41
20	597783	4·88	962945	·91	634838	5·79	365162	40
21	9·598075	4·87	9·962890	·91	9·635185	5·78	10·364815	39
22	598368	4·87	962836	·91	635532	5·78	364468	38
23	598660	4·87	962781	·91	635879	5·78	364121	37
24	598952	4·86	962727	·91	636226	5·77	363774	36
25	599244	4·86	962672	·91	636572	5·77	363428	35
26	599536	4·85	962617	·91	636919	5·77	363081	34
27	599827	4·85	962562	·91	637265	5·77	362735	33
28	600118	4·85	962508	·91	637611	5·76	362389	32
29	600409	4·84	962453	·91	637956	5·76	362044	31
30	600700	4·84	962398	·92	638302	5·76	361698	30
31	9·600990	4·84	9·962343	·92	9·638647	5·75	10·361353	29
32	601280	4·83	962288	·92	638992	5·75	361008	28
33	601570	4·83	962233	·92	639337	5·75	360663	27
34	601860	4·82	962178	·92	639682	5·74	360318	26
35	602150	4·82	962123	·92	640027	5·74	359973	25
36	602439	4·82	962067	·92	640371	5·74	359629	24
37	602728	4·81	962012	·92	640716	5·73	359284	23
38	603017	4·81	961957	·92	641060	5·73	358940	22
39	603305	4·81	961902	·92	641404	5·73	358596	21
40	603594	4·80	961846	·92	641747	5·72	358253	20
41	9·603882	4·80	9·961791	·92	9·642091	5·72	10·357909	19
42	604170	4·79	961735	·92	642434	5·72	357566	18
43	604457	4·79	961680	·92	642777	5·72	357223	17
44	604745	4·79	961624	·93	643120	5·71	356880	16
45	605032	4·78	961569	·93	643463	5·71	356537	15
46	605319	4·78	961513	·93	643806	5·71	356194	14
47	605606	4·78	961458	·93	644148	5·70	355852	13
48	605892	4·77	961402	·93	644490	5·70	355510	12
49	606179	4·77	961346	·93	644832	5·70	355168	11
50	606465	4·76	961290	·93	645174	5·69	354826	10
51	9·606751	4·76	9·961235	·93	9·645516	5·69	10·354484	9
52	607036	4·76	961179	·93	645857	5·69	354143	8
53	607322	4·75	961123	·93	646199	5·69	353801	7
54	607607	4·75	961067	·93	646540	5·68	353460	6
55	607892	4·74	961011	·93	646881	5·68	353119	5
56	608177	4·74	960955	·93	647222	5·68	352778	4
57	608461	4·74	960899	·93	647562	5·67	352438	3
58	608745	4·73	960843	·94	647903	5·67	352097	2
59	609029	4·73	960786	·94	648243	5·67	351757	1
60	609313	4·73	960730	·94	648583	5·66	351417	0
	Cosine	D.	Sine	66°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·609313	4·73	9·960730	·94	9·648583	5·66	10·351417	60
1	609597	4·72	960674	·94	648923	5·66	351077	59
2	609880	4·72	960618	·94	649263	5·66	350737	58
3	610164	4·72	960561	·94	649602	5·66	350398	57
4	610447	4·71	960505	·94	649942	5·65	350058	56
5	610729	4·71	960448	·94	650281	5·65	349719	55
6	611012	4·70	960392	·94	650620	5·65	349380	54
7	611294	4·70	960335	·94	650959	5·64	349041	53
8	611576	4·70	960279	·94	651297	5·64	348703	52
9	611858	4·69	960222	·94	651636	5·64	348364	51
10	612140	4·69	960165	·94	651974	5·63	348026	50
11	9·612421	4·69	9·960109	·95	9·652312	5·63	10·347688	49
12	612702	4·68	960052	·95	652650	5·63	347350	48
13	612983	4·68	959995	·95	652988	5·63	347012	47
14	613264	4·67	959938	·95	653326	5·62	346674	46
15	613545	4·67	959882	·95	653663	5·62	346337	45
16	613825	4·67	959825	·95	654000	5·62	346000	44
17	614105	4·66	959768	·95	654337	5·61	345663	43
18	614385	4·66	959711	·95	654674	5·61	345326	42
19	614665	4·66	959654	·95	655011	5·61	344989	41
20	614944	4·65	959596	·95	655348	5·61	344652	40
21	9·615223	4·65	9·959539	·95	9·655684	5·60	10·344316	39
22	615502	4·65	959482	·95	656020	5·60	343980	38
23	615781	4·64	959425	·95	656356	5·60	343644	37
24	616060	4·64	959368	·95	656692	5·59	343308	36
25	616338	4·64	959310	·96	657028	5·59	342972	35
26	616616	4·63	959253	·96	657364	5·59	342636	34
27	616894	4·63	959195	·96	657699	5·59	342301	33
28	617172	4·62	959138	·96	658034	5·58	341966	32
29	617450	4·62	959081	·96	658369	5·58	341631	31
30	617727	4·62	959023	·96	658704	5·58	341296	30
31	9·618004	4·61	9·958965	·96	9·659039	5·58	10·340961	29
32	618281	4·61	958908	·96	659373	5·57	340627	28
33	618558	4·61	958850	·96	659708	5·57	340292	27
34	618834	4·60	958792	·96	660042	5·57	339958	26
35	619110	4·60	958734	·96	660376	5·57	339624	25
36	619386	4·60	958677	·96	660710	5·56	339290	24
37	619662	4·59	958619	·96	661043	5·56	338957	23
38	619938	4·59	958561	·96	661377	5·56	338623	22
39	620213	4·59	958503	·97	661710	5·55	338290	21
40	620488	4·58	958445	·97	662043	5·55	337957	20
41	9·620763	4·58	9·958387	·97	9·662376	5·55	10·337624	19
42	621038	4·57	958329	·97	662709	5·54	337291	18
43	621313	4·57	958271	·97	663042	5·54	336958	17
44	621587	4·57	958213	·97	663375	5·54	336625	16
45	621861	4·56	958154	·97	663707	5·54	336293	15
46	622135	4·56	958096	·97	664039	5·53	335961	14
47	622409	4·56	958038	·97	664371	5·53	335629	13
48	622682	4·55	957979	·97	664703	5·53	335297	12
49	622956	4·55	957921	·97	665035	5·53	334965	11
50	623229	4·55	957863	·97	665366	5·52	334634	10
51	9·623502	4·54	9·957804	·97	9·665697	5·52	10·334303	9
52	623774	4·54	957746	·98	666029	5·52	333971	8
53	624047	4·54	957687	·98	666360	5·51	333640	7
54	624319	4·53	957628	·98	666691	5·51	333309	6
55	624591	4·53	957570	·98	667021	5·51	332979	5
56	624863	4·53	957511	·98	667352	5·51	332648	4
57	625135	4·52	957452	·98	667682	5·50	332318	3
58	625406	4·52	957393	·98	668013	5·50	331987	2
59	625677	4·52	957335	·98	668343	5·50	331657	1
60	625948	4·51	957276	·98	668672	5·50	331328	0
	Cosine	D.	Sine	65°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·625948	4·51	9·957276	·98	9·668673	5·50	10·331327	60
1	626219	4·51	957217	·98	669002	5·49	330998	59
2	626490	4·51	957158	·98	669332	5·49	330668	58
3	626760	4·50	957099	·98	669661	5·49	330339	57
4	627030	4·50	957040	·98	669991	5·48	330009	56
5	627300	4·50	956981	·98	670320	5·48	329680	55
6	627570	4·49	956921	·99	670649	5·48	329351	54
7	627840	4·49	956862	·99	670977	5·48	329023	53
8	628109	4·49	956803	·99	671306	5·47	328694	52
9	628378	4·48	956744	·99	671634	5·47	328366	51
10	628647	4·48	956684	·99	671963	5·47	328037	50
11	9·628916	4·47	9·956625	·99	9·672291	5·47	10·327709	49
12	629185	4·47	956566	·99	672619	5·46	327381	48
13	629453	4·47	956506	·99	672947	5·46	327053	47
14	629721	4·46	956447	·99	673274	5·46	326726	46
15	629989	4·46	956387	·99	673602	5·46	326398	45
16	630257	4·46	956327	·99	673929	5·45	326071	44
17	630524	4·46	956268	·99	674257	5·45	325743	43
18	630792	4·45	956208	1·00	674584	5·45	325416	42
19	631059	4·45	956148	1·00	674910	5·44	325090	41
20	631326	4·45	956089	1·00	675237	5·44	324763	40
21	9·631593	4·44	9·956029	1·00	9·675564	5·44	10·324436	39
22	631859	4·44	955969	1·00	675890	5·44	324110	38
23	632125	4·44	955909	1·00	676216	5·43	323784	37
24	632392	4·43	955849	1·00	676543	5·43	323457	36
25	632658	4·43	955789	1·00	676869	5·43	323131	35
26	632923	4·43	955729	1·00	677194	5·43	322806	34
27	633189	4·42	955669	1·00	677520	5·42	322480	33
28	633454	4·42	955609	1·00	677846	5·42	322154	32
29	633719	4·42	955548	1·00	678171	5·42	321829	31
30	633984	4·41	955488	1·00	678496	5·42	321504	30
31	9·634249	4·41	9·955428	1·01	9·678821	5·41	10·321179	29
32	634514	4·40	955368	1·01	679146	5·41	320854	28
33	634778	4·40	955307	1·01	679471	5·41	320529	27
34	635042	4·40	955247	1·01	679795	5·41	320205	26
35	635306	4·39	955186	1·01	680120	5·40	319880	25
36	635570	4·39	955126	1·01	680444	5·40	319556	24
37	635834	4·39	955065	1·01	680768	5·40	319232	23
38	636097	4·38	955005	1·01	681092	5·40	318908	22
39	636360	4·38	954944	1·01	681416	5·39	318584	21
40	636623	4·38	954883	1·01	681740	5·39	318260	20
41	9·636886	4·37	9·954823	1·01	9·682063	5·39	10·317937	19
42	637148	4·37	954762	1·01	682387	5·39	317613	18
43	637411	4·37	954701	1·91	682710	5·38	317290	17
44	637673	4·37	954640	1·01	683033	5·38	316967	16
45	637935	4·36	954579	1·[illegible]	683356	5·38	316644	15
46	638197	4·36	954518	1·02	683679	5·38	316321	14
47	638458	4·36	954457	1·02	684001	5·37	315999	13
48	638720	4·35	954396	1·02	684324	5·37	315676	12
49	638981	4·35	954335	1·02	684646	5·37	315354	11
50	639242	4·35	954274	1·02	684968	5·37	315032	10
51	9·639503	4·34	9·954213	1·02	9·685290	5·36	10·314710	9
52	639764	4·34	954152	1·02	685612	5·36	314388	8
53	640024	4·34	954090	1·02	685934	5·36	314066	7
54	640284	4·33	954029	1·02	686255	5·36	313745	6
55	640544	4·33	953968	1·02	686577	5·35	313423	5
56	640804	4·33	953906	1·02	686898	5·35	313102	4
57	641064	4·32	953845	1·02	687219	5·35	312781	3
58	641324	4·32	953783	1·02	687540	5·35	312460	2
59	641584	4·32	953722	1·03	687861	5·34	312139	1
60	641842	4·31	953660	1·03	688182	5·34	311818	0
	Cosine	D.	Sine	64°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·641842	4·31	9·953660	1·03	9·688182	5·34	10·311818	60
1	642101	4·31	953599	1·03	688502	5·34	311498	59
2	642360	4·31	953537	1·03	688823	5·34	311177	58
3	642618	4·30	953475	1·03	689143	5·33	310857	57
4	642877	4·30	953413	1·03	689463	5·33	310537	56
5	643135	4·30	953352	1·03	689783	5·33	310217	55
6	643393	4·30	953290	1·03	690103	5·33	309897	54
7	643650	4·29	953228	1·03	690423	5·33	309577	53
8	643908	4·29	953166	1·03	690742	5·32	309258	52
9	644165	4·29	953104	1·03	691062	5·32	308938	51
10	644423	4·28	953042	1·03	691381	5·32	308619	50
11	9·644680	4·28	9·952980	1·04	9·691700	5·31	10·308300	49
12	644936	4·28	952918	1·04	692019	5·31	307981	48
13	645193	4·27	952855	1·04	692338	5·31	307662	47
14	645450	4·27	952793	1·04	692656	5·31	307344	46
15	645706	4·27	952731	1·04	692975	5·31	307025	45
16	645962	4·26	952669	1·04	693293	5·30	306707	44
17	646218	4·26	952606	1·04	693612	5·30	306388	43
18	646474	4·26	952544	1·04	693930	5·30	306070	42
19	646729	4·25	952481	1·04	694248	5·30	305752	41
20	646984	4·25	952419	1·04	694566	5·29	305434	40
21	9·647240	4·25	9·952356	1·04	9·694883	5·29	10·305117	39
22	647494	4·24	952294	1·04	695201	5·29	304799	38
23	647749	4·24	952231	1·04	695518	5·29	304482	37
24	648004	4·24	952168	1·05	695836	5·29	304164	36
25	648258	4·24	952106	1·05	696153	5·28	303847	35
26	648512	4·23	952043	1·05	696470	5·28	303530	34
27	648766	4·23	951980	1·05	696787	5·28	303213	33
28	649020	4·23	951917	1·05	697103	5·28	302897	32
29	649274	4·22	951854	1·05	697420	5·27	302580	31
30	649527	4·22	951791	1·05	697736	5·27	302264	30
31	9·649781	4·22	9·951728	1·05	9·698053	5·27	10·301947	29
32	650034	4·22	951665	1·05	698369	5·27	301631	28
33	650287	4·21	951602	1·05	698685	5·26	301315	27
34	650539	4·21	951539	1·05	699001	5·26	300999	26
35	650792	4·21	951476	1·05	699316	5·26	300684	25
36	651044	4·20	951412	1·05	699632	5·26	300368	24
37	651297	4·20	951349	1·06	699947	5·26	300053	23
38	651549	4·20	951286	1·06	700263	5·25	299737	22
39	651800	4·19	951222	1·06	700578	5·25	299422	21
40	652052	4·19	951159	1·06	700893	5·25	299107	20
41	9·652304	4·19	9·951096	1·06	9·701208	5·24	10·298792	19
42	652555	4·18	951032	1·06	701523	5·24	298477	18
43	652806	4·18	950968	1·06	701837	5·24	298163	17
44	653057	4·18	950905	1·06	702152	5·24	297848	16
45	653308	4·18	950841	1·06	702466	5·24	297534	15
46	653558	4·17	950778	1·06	702780	5·23	297220	14
47	653808	4·17	950714	1·06	703095	5·23	296905	13
48	654059	4·17	950650	1·06	703409	5·23	296591	12
49	654309	4·16	950586	1·06	703723	5·23	296277	11
50	654558	4·16	950522	1·07	704036	5·22	295964	10
51	9·654808	4·16	9·950458	1·07	9·704350	5·22	10·295650	9
52	655058	4·16	950394	1·07	704663	5·22	295337	8
53	655307	4·15	950330	1·07	704977	5·22	295023	7
54	655556	4·15	950266	1·07	705290	5·22	294710	6
55	655805	4·15	950202	1·07	705603	5·21	294397	5
56	656054	4·14	950138	1·07	705916	5·21	294084	4
57	656302	4·14	950074	1·07	706228	5·21	293772	3
58	656551	4·14	950010	1·07	706541	5·21	293459	2
59	656799	4·13	949945	1·07	706854	5·21	293146	1
60	657047	4·13	949881	1·07	707166	5·20	292834	0
	Cosine	D.	Sine	63°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·657047	4·13	9·949881	1·07	9·707166	5·20	10·292834	60
1	657295	4·13	949816	1·07	707478	5·20	292522	59
2	657542	4·12	949752	1·07	707790	5·20	292210	58
3	657790	4·12	949688	1·08	708102	5·20	291898	57
4	658037	4·12	949623	1·08	708414	5·19	291586	56
5	658284	4·12	949558	1·08	708726	5·19	291274	55
6	658531	4·11	949494	1·08	709037	5·19	290963	54
7	658778	4·11	949429	1·08	709349	5·19	290651	53
8	659025	4·11	949364	1·08	709660	5·19	290340	52
9	659271	4·10	949300	1·08	709971	5·18	290029	51
10	659517	4·10	949235	1·08	710282	5·18	289718	50
11	9·659763	4·10	9·949170	1·08	9·710593	5·18	10·289407	49
12	660009	4·09	949105	1·08	710904	5·18	289096	48
13	660255	4·09	949040	1·08	711215	5·18	288785	47
14	660501	4·09	948975	1·08	711525	5·17	288475	46
15	660746	4·09	948910	1·08	711836	5·17	288164	45
16	660991	4·08	948845	1·08	712146	5·17	287854	44
17	661236	4·08	948780	1·09	712456	5·17	287544	43
18	661481	4·08	948715	1·09	712766	5·16	287234	42
19	661726	4·07	948650	1·09	713076	5·16	286924	41
20	661970	4·07	948584	1·09	713386	5·16	286614	40
21	9·662214	4·07	9·948519	1·09	9·713696	5·16	10·286304	39
22	662459	4·07	948454	1·09	714005	5·16	285995	38
23	662703	4·06	948388	1·09	714314	5·15	285686	37
24	662946	4·06	948323	1·09	714624	5·15	285376	36
25	663190	4·06	948257	1·09	714933	5·15	285067	35
26	663433	4·05	948192	1·09	715242	5·15	284758	34
27	663677	4·05	948126	1·09	715551	5·14	284449	33
28	663920	4·05	948060	1·09	715860	5·14	284140	32
29	664163	4·05	947995	1·10	716168	5·14	283832	31
30	664406	4·04	947929	1·10	716477	5·14	283523	30
31	9·664648	4·04	9·947863	1·10	9·716785	5·14	10·283215	29
32	664891	4·04	947797	1·10	717093	5·13	282907	28
33	665133	4·03	947731	1·10	717401	5·13	282599	27
34	665375	4·03	947665	1·10	717709	5·13	282291	26
35	665617	4·03	947600	1·10	718017	5·13	281983	25
36	665859	4·02	947533	1·10	718325	5·13	281670	24
37	666100	4·02	947467	1·10	718633	5·12	281367	23
38	666342	4·02	947401	1·10	718940	5·12	281060	22
39	666583	4·02	947335	1·10	719248	5·12	280752	21
40	666824	4·01	947269	1·10	719555	5·12	280445	20
41	9·667065	4·01	9·947203	1·10	9·719862	5·12	10·280138	19
42	667305	4·01	947136	1·11	720169	5·11	279831	18
43	667546	4·01	947070	1·11	720476	5·11	279524	17
44	667786	4·00	947004	1·11	720783	5·11	279217	16
45	668027	4·00	946937	1·11	721089	5·11	278911	15
46	668267	4·00	946871	1·11	721396	5·11	278604	14
47	668506	3·99	946804	1·11	721702	5·10	278298	13
48	668746	3·99	946738	1·11	722009	5·10	277991	12
49	668986	3·99	946671	1·11	722315	5·10	277685	11
50	669225	3·99	946604	1·11	722621	5·10	277379	10
51	9·669464	3·98	9·946538	1·11	9·722927	5·10	10·277073	9
52	669703	3·98	946471	1·11	723232	5·09	276768	8
53	669942	3·98	946404	1·11	723538	5·09	276462	7
54	670181	3·97	946337	1·11	723844	5·09	276156	6
55	670419	3·97	946270	1·12	724149	5·09	275851	5
56	670658	3·97	946203	1·12	724454	5·09	275546	4
57	670896	3·97	946136	1·12	724759	5·08	275241	3
58	671134	3·96	946069	1·12	725065	5·08	274935	2
59	671372	3·96	946002	1·12	725369	5·08	274631	1
60	671609	3·96	945935	1·12	725674	5·08	274326	0
	Cosine	D.	Sine	62°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·671609	3·96	9·945935	1·12	9·725674	5·08	10·274326	60
1	671847	3·95	945868	1·12	725979	5·08	274021	59
2	672084	3·95	945800	1·12	726284	5·07	273716	58
3	672321	3·95	945733	1·12	726588	5·07	273412	57
4	672558	3·95	945666	1·12	726892	5·07	273108	56
5	672795	3·94	945598	1·12	727197	5·07	272803	55
6	673032	3·94	945531	1·12	727501	5·07	272499	54
7	673268	3·94	945464	1·13	727805	5·06	272195	53
8	673505	3·94	945396	1·13	728109	5·06	271891	52
9	673741	3·93	945328	1·13	728412	5·06	271588	51
10	673977	3·93	945261	1·13	728716	5·06	271284	50
11	9·674213	3·93	9·945193	1·13	9·729020	5·06	10·270980	49
12	674448	3·92	945125	1·13	729323	5·05	270677	48
13	674684	3·92	945058	1·13	729626	5·05	270374	47
14	674919	3·92	944990	1·13	729929	5·05	270071	46
15	675155	3·92	944922	1·13	730233	5·05	269767	45
16	675390	3·91	944854	1·13	730535	5·05	269465	44
17	675624	3·91	944786	1·13	730838	5·04	269162	43
18	675859	3·91	944718	1·13	731141	5·04	268859	42
19	676094	3·91	944650	1·13	731444	5·04	268556	41
20	676328	3·90	944582	1·14	731746	5·04	268254	40
21	9·676562	3·90	9·944514	1·14	9·732048	5·04	10·267952	39
22	676796	3·90	944446	1·14	732351	5·03	267649	38
23	677030	3·90	944377	1·14	732653	5·03	267347	37
24	677264	3·89	944309	1·14	732955	5·03	267045	36
25	677498	3·89	944241	1·14	733257	5·03	266743	35
26	677731	3·89	944172	1·14	733558	5·03	266442	34
27	677964	3·88	944104	1·14	733860	5·02	266140	33
28	678197	3·88	944036	1·14	734162	5·02	265838	32
29	678430	3·88	943967	1·14	734463	5·02	265537	31
30	678663	3·88	943899	1·14	734764	5·02	265236	30
31	9·678895	3·87	9·943830	1·14	9·735066	5·02	10·264934	29
32	679128	3·87	943761	1·14	735367	5·02	264633	28
33	679360	3·87	943693	1·15	735668	5·01	264332	27
34	679592	3·87	943624	1·15	735969	5·01	264031	26
35	679824	3·86	943555	1·15	736269	5·01	263731	25
36	680056	3·86	943486	1·15	736570	5·01	263430	24
37	680288	3·86	943417	1·15	736871	5·01	263129	23
38	680519	3·85	943348	1·15	737171	5·00	262829	22
39	680750	3·85	943279	1·15	737471	5·00	262529	21
40	680982	3·85	943210	1·15	737771	5·00	262229	20
41	9·681213	3·85	9·943141	1·15	9·738071	5·00	10·261929	19
42	681443	3·84	943072	1·15	738371	5·00	261629	18
43	681674	3·84	943003	1·15	738671	4·99	261329	17
44	681905	3·84	942934	1·15	738971	4·99	261029	16
45	682135	3·84	942864	1·15	739271	4·99	260729	15
46	682365	3·83	942795	1·16	739570	4·99	260430	14
47	682595	3·83	942726	1·16	739870	4·99	260130	13
48	682825	3·83	942656	1·16	740169	4·99	259831	12
49	683055	3·83	942587	1·16	740468	4·98	259532	11
50	683284	3·82	942517	1·16	740767	4·98	259233	10
51	9·683514	3·82	9·942448	1·16	9·741066	4·98	10·258934	9
52	683743	3·82	942378	1·16	741365	4·98	258635	8
53	683972	3·82	942308	1·16	741664	4·98	258336	7
54	684201	3·81	942239	1·16	741962	4·97	258038	6
55	684430	3·81	942169	1·16	742261	4·97	257739	5
56	684658	3·81	942099	1·16	742559	4·97	257441	4
57	684887	3·80	942029	1·16	742858	4·97	257142	3
58	685115	3·80	941959	1·16	743156	4·97	256844	2
59	685343	3·80	941889	1·17	743454	4·97	256546	1
60	685571	3·80	941819	1·17	743752	4·96	256248	0
	Cosine	D.	Sine	61°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·685571	3·80	9·941819	1·17	9·743752	4·96	10·256248	60
1	685799	3·79	941749	1·17	744050	4·96	255950	59
2	686027	3·79	941679	1·17	744348	4·96	255652	58
3	686254	3·79	941609	1·17	744645	4·96	255355	57
4	686482	3·79	941539	1·17	744943	4·96	255057	56
5	686709	3·78	941469	1·17	745240	4·96	254760	55
6	686936	3·78	941398	1·17	745538	4·95	254462	54
7	687163	3·78	941328	1·17	745835	4·95	254165	53
8	687389	3·78	941258	1·17	746132	4·95	253868	52
9	687616	3·77	941187	1·17	746429	4·95	253571	51
10	687843	3·77	941117	1·17	746726	4·95	253274	50
11	9·688069	3·77	9·941046	1·18	9·747023	4·94	10·252977	49
12	688295	3·77	940975	1·18	747319	4·94	252681	48
13	688521	3·76	940905	1·18	747616	4·94	252384	47
14	688747	3·76	940834	1·18	747913	4·94	252087	46
15	688972	3·76	940763	1·18	748209	4·94	251791	45
16	689198	3·76	940693	1·18	748505	4·93	251495	44
17	689423	3·75	940622	1·18	748801	4·93	251199	43
18	689648	3·75	940551	1·18	749097	4·93	250903	42
19	689873	3·75	940480	1·18	749393	4·93	250607	41
20	690098	3·75	940409	1·18	749689	4·93	250311	40
21	9·690323	3·74	9·940338	1·18	9·749985	4·93	10·250015	39
22	690548	3·74	940267	1·18	750281	4·92	249719	38
23	690772	3·74	940196	1·18	750576	4·92	249424	37
24	690996	3·74	940125	1·19	750872	4·92	249128	36
25	691220	3·73	940054	1·19	751167	4·92	248833	35
26	691444	3·73	939982	1·19	751462	4·92	248538	34
27	691668	3·73	939911	1·19	751757	4·92	248243	33
28	691892	3·73	939840	1·19	752052	4·91	247948	32
29	692115	3·72	939768	1·19	752347	4·91	247653	31
30	692339	3·72	939697	1·19	752642	4·91	247358	30
31	9·692562	3·72	9·939625	1·19	9·752937	4·91	10·247063	29
32	692785	3·71	939554	1·19	753231	4·91	246769	28
33	693008	3·71	939482	1·19	753526	4·91	246474	27
34	693231	3·71	939410	1·19	753820	4·90	246180	26
35	693453	3·71	939339	1·19	754115	4·90	245885	25
36	693676	3·70	939267	1·20	754409	4·90	245591	24
37	693898	3·70	939195	1·20	754703	4·90	245297	23
38	694120	3·70	939123	1·20	754997	4·90	245003	22
39	694342	3·70	939052	1·20	755291	4·90	244709	21
40	694564	3·69	938980	1·20	755585	4·89	244415	20
41	9·694786	3·69	9·938908	1·20	9·755878	4·89	10·244122	19
42	695007	3·69	938836	1·20	756172	4·89	243828	18
43	695229	3·69	938763	1·20	756465	4·89	243535	17
44	695450	3·68	938691	1·20	756759	4·89	243241	16
45	695671	3·68	938619	1·20	757052	4·89	242948	15
46	695892	3·68	938547	1·20	757345	4·88	242655	14
47	696113	3·68	938475	1·20	757638	4·88	242362	13
48	696334	3·67	938402	1·21	757931	4·88	242069	12
49	696554	3·67	938330	1·21	758224	4·88	241776	11
50	696775	3·67	938258	1·21	758517	4·88	241483	10
51	9·696995	3·67	9·938185	1·21	9·758810	4·88	10·241190	9
52	697215	3·66	938113	1·21	759102	4·87	240898	8
53	697435	3·66	938040	1·21	759395	4·87	240605	7
54	697654	3·66	937967	1·21	759687	4·87	240313	6
55	697874	3·66	937895	1·21	759979	4·87	240021	5
56	698094	3·65	937822	1·21	760272	4·87	239728	4
57	698313	3·65	937749	1·21	760564	4·87	239436	3
58	698532	3·65	937676	1·21	760856	4·86	239144	2
59	698751	3·65	937604	1·21	761148	4·86	238852	1
60	698970	3·64	937531	1·21	761439	4·86	238561	0
	Cosine	D.	Sine	60°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·698970	3·64	9·937531	1·21	9·761439	4·86	10·238561	60
1	699189	3·64	937458	1·22	761731	4·86	238269	59
2	699407	3·64	937385	1·22	762023	4·86	237977	58
3	699626	3·64	937312	1·22	762314	4·86	237686	57
4	699844	3·63	937238	1·22	762606	4·85	237394	56
5	700062	3·63	937165	1·22	762897	4·85	237103	55
6	700280	3·63	937092	1·22	763188	4·85	236812	54
7	700498	3·63	937019	1·22	763479	4·85	236521	53
8	700716	3·63	936946	1·22	763770	4·85	236230	52
9	700933	3·62	936872	1·22	764061	4·85	235939	51
10	701151	3·62	936799	1·22	764352	4·84	235648	50
11	9·701368	3·62	9·936725	1·22	9·764643	4·84	10·235357	49
12	701585	3·62	936652	1·23	764933	4·84	235067	48
13	701802	3·61	936578	1·23	765224	4·84	234776	47
14	702019	3·61	936505	1·23	765514	4·84	234486	46
15	702236	3·61	936431	1·23	765805	4·84	234195	45
16	702452	3·61	936357	1·23	766095	4·84	233905	44
17	702669	3·60	936284	1·23	766385	4·83	233615	43
18	702885	3·60	936210	1·23	766675	4·83	233325	42
19	703101	3·60	936136	1·23	766965	4·83	233035	41
20	703317	3·60	936062	1·23	767255	4·83	232745	40
21	9·703533	3·59	9·935988	1·23	9·767545	4·83	10·232455	39
22	703749	3·59	935914	1·23	767834	4·83	232166	38
23	703964	3·59	935840	1·23	768124	4·82	231876	37
24	704179	3·59	935766	1·24	768413	4·82	231587	36
25	704395	3·59	935692	1·24	768703	4·82	231297	35
26	704610	3·58	935618	1·24	768992	4·82	231008	34
27	704825	3·58	935543	1·24	769281	4·82	230719	33
28	705040	3·58	935469	1·24	769570	4·82	230430	32
29	705254	3·58	935395	1·24	769860	4·81	230140	31
30	705469	3·57	935320	1·24	770148	4·81	229852	30
31	9·705683	3·57	9·935246	1·24	9·770437	4·81	10·229563	29
32	705898	3·57	935171	1·24	770726	4·81	229274	28
33	706112	3·57	935097	1·24	771015	4·81	228985	27
34	706326	3·56	935022	1·24	771303	4·81	228697	26
35	706539	3·56	934948	1·24	771592	4·81	228408	25
36	706753	3·56	934873	1·24	771880	4·80	228120	24
37	706967	3·56	934798	1·25	772168	4·80	227832	23
38	707180	3·55	934723	1·25	772457	4·80	227543	22
39	707393	3·55	934649	1·25	772745	4·80	227255	21
40	707606	3·55	934574	1·25	773033	4·80	226967	20
41	9·707819	3·55	9·934499	1·25	9·773321	4·80	10·226679	19
42	708032	3·54	934424	1·25	773608	4·79	226392	18
43	708245	3·54	934349	1·25	773896	4·79	226104	17
44	708458	3·54	934274	1·25	774184	4·79	225816	16
45	708670	3·54	934199	1·25	774471	4·79	225529	15
46	708882	3·53	934123	1·25	774759	4·79	225241	14
47	709094	3·53	934048	1·25	775046	4·79	224954	13
48	709306	3·53	933973	1·25	775333	4·79	224667	12
49	709518	3·53	933898	1·26	775621	4·78	224379	11
50	709730	3·53	933822	1·26	775908	4·78	224092	10
51	9·709941	3·52	9·933747	1·26	9·776195	4·78	10·223805	9
52	710153	3·52	933671	1·26	776482	4·78	223518	8
53	710364	3·52	933596	1·26	776769	4·78	223231	7
54	710575	3·52	933520	1·26	777055	4·78	222945	6
55	710786	3·51	933445	1·26	777342	4·78	222658	5
56	710997	3·51	933369	1·26	777628	4·77	222372	4
57	711208	3·51	933293	1·26	777915	4·77	222085	3
58	711419	3·51	933217	1·26	778201	4·77	221799	2
59	711629	3·50	933141	1·26	778487	4·77	221512	1
60	711839	3·50	933066	1·26	778774	4·77	221226	0
	Cosine	D.	Sine	59°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·711839	3·50	9·933066	1·26	9·778774	4·77	10·221226	60
1	712050	3·50	932990	1·27	779060	4·77	220940	59
2	712260	3·50	932914	1·27	779346	4·76	220654	58
3	712469	3·49	932838	1·27	779632	4·76	220368	57
4	712679	3·49	932762	1·27	779918	4·76	220082	56
5	712889	3·49	932685	1·27	780203	4·76	219797	55
6	713098	3·49	932609	1·27	780489	4·76	219511	54
7	713308	3·49	932533	1·27	780775	4·76	219225	53
8	713517	3·48	932457	1·27	781060	4·76	218940	52
9	713726	3·48	932380	1·27	781346	4·75	218654	51
10	713935	3·48	932304	1·27	781631	4·75	218369	50
11	9·714144	3·48	9·932228	1·27	9·781916	4·75	10·218084	49
12	714352	3·47	932151	1·27	782201	4·75	217799	48
13	14561	3·47	932075	1·28	782486	4·75	217514	47
14	714769	3·47	931998	1·28	782771	4·75	217229	46
15	714978	3·47	931921	1·28	783056	4·75	216944	45
16	715186	3·47	931845	1·28	783341	4·75	216659	44
17	715394	3·46	931768	1·28	783626	4·74	216374	43
18	715602	3·46	931691	1·28	783910	4·74	216090	42
19	715809	3·46	931614	1·28	784195	4·74	215805	41
20	716017	3·46	931537	1·28	784479	4·74	215521	40
21	9·716224	3·45	9·931460	1·28	9·784764	4·74	10·215236	39
22	716432	3·45	931383	1·28	785048	4·74	214952	38
23	716639	3·45	931306	1·28	785332	4·73	214668	37
24	716846	3·45	931229	1·29	785616	4·73	214384	36
25	717053	3·45	931152	1·29	785900	4·73	214100	35
26	717259	3·44	931075	1·29	786184	4·73	213816	34
27	717466	3·44	930998	1·29	786468	4·73	213532	33
28	717673	3·44	930921	1·29	786752	4·73	213248	32
29	717879	3·44	930843	1·29	787036	4·73	212964	31
30	718085	3·43	930766	1·29	787319	4·72	212681	30
31	9·718291	3·43	9·930688	1·29	9·787603	4·72	10·212397	29
32	718497	3·43	930611	1·29	787886	4·72	212114	28
33	718703	3·43	930533	1·29	788170	4·72	211830	27
34	718909	3·43	930456	1·29	788453	4·72	211547	26
35	719114	3·42	930378	1·29	788736	4·72	211264	25
36	719320	3·42	930300	1·30	789019	4·72	210981	24
37	719525	3·42	930223	1·30	789302	4·71	210698	23
38	719730	3·42	930145	1·30	789585	4·71	210415	22
39	719935	3·41	930067	1·30	789868	4·71	210132	21
40	720140	3·41	929989	1·30	790151	4·71	209849	20
41	9·720345	3·41	9·929911	1·30	9·790433	4·71	10·209567	19
42	720549	3·41	929833	1·30	790716	4·71	209284	18
43	720754	3·40	929755	1·30	790999	4·71	209001	17
44	720958	3·40	929677	1·30	791281	4·71	208719	16
45	721162	3·40	929599	1·30	791563	4·70	208437	15
46	721366	3·40	929521	1·30	791846	4·70	208154	14
47	721570	3·40	929442	1·30	792128	4·70	207872	13
48	721774	3·39	929364	1·31	792410	4·70	207590	12
49	721978	3·39	929286	1·31	792692	4·70	207308	11
50	722181	3·39	929207	1·31	792974	4·70	207026	10
51	9·722385	3·39	9·929129	1·31	9·793256	4·70	10·206744	9
52	722588	3·39	929050	1·31	793538	4·69	206462	8
53	722791	3·38	928972	1·31	793819	4·69	206181	7
54	722994	3·38	928893	1·31	794101	4·69	205899	6
55	723197	3·38	928815	1·31	794383	4·69	205617	5
56	723400	3·38	928736	1·31	794664	4·69	205336	4
57	723603	3·37	928657	1·31	794945	4·69	205055	3
58	723805	3·37	928578	1·31	795227	4·69	204773	2
59	724007	3·37	928499	1·31	795508	4·68	204492	1
60	724210	3·37	928420	1·31	795789	4·68	204211	0
	Cosine	D.	Sine	58°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·724210	3·37	9·928420	1·32	9·795789	4·68	10·204211	60
1	724412	3·37	928342	1·32	796070	4·68	203930	59
2	724614	3·36	928263	1·32	796351	4·68	203649	58
3	724816	3·36	928183	1·32	796632	4·68	203368	57
4	725017	3·36	928104	1·32	796913	4·68	203087	56
5	725219	3·36	928025	1·32	797194	4·68	202806	55
6	725420	3·35	927946	1·32	797475	4·68	202525	54
7	725622	3·35	927867	1·32	797755	4·68	202245	53
8	725823	3·35	927787	1·32	798036	4·67	201964	52
9	726024	3·35	927708	1·32	798316	4·67	201684	51
10	726225	3·35	927629	1·32	798596	4·67	201404	50
11	9·726426	3·34	9·927549	1·32	9·798877	4·67	10·201123	49
12	726626	3·34	927470	1·33	799157	4·67	200843	48
13	726827	3·34	927390	1·33	799437	4·67	200563	47
14	727027	3·34	927310	1·33	799717	4·67	200283	46
15	727228	3·34	927231	1·33	799997	4·66	200003	45
16	727428	3·33	927151	1·33	800277	4·66	199723	44
17	727628	3·33	927071	1·33	800557	4·66	199443	43
18	727828	3·33	926991	1·33	800836	4·66	199164	42
19	728027	3·33	926911	1·33	801116	4·66	198884	41
20	728227	3·33	926831	1·33	801396	4·66	198604	40
21	9·728427	3·32	9·926751	1·33	9·801675	4·66	10·198325	39
22	728626	3·32	926671	1·33	801955	4·66	198045	38
23	728825	3·32	926591	1·33	802234	4·65	197766	37
24	729024	3·32	926511	1·34	802513	4·65	197487	36
25	729223	3·31	926431	1·34	802792	4·65	197208	35
26	729422	3·31	926351	1·34	803072	4·65	196928	34
27	729621	3·31	926270	1·34	803351	4·65	196649	33
28	729820	3·31	926190	1·34	803630	4·65	196370	32
29	730018	3·30	926110	1·34	803908	4·65	196092	31
30	730216	3·30	926029	1·34	804187	4·65	195813	30
31	9·730415	3·30	9·925949	1·34	9·804466	4·64	10·195534	29
32	730613	3·30	925868	1·34	804745	4·64	195255	28
33	730811	3·30	925788	1·34	805023	4·64	194977	27
34	731009	3·29	925707	1·34	805302	4·64	194698	26
35	731206	3·29	925626	1·34	805580	4·64	194420	25
36	731404	3·29	925545	1·35	805859	4·64	194141	24
37	731602	3·29	925465	1·35	806137	4·64	193863	23
38	731799	3·29	925384	1·35	806415	4·63	193585	22
39	731996	3·28	925303	1·35	806693	4·63	193307	21
40	732193	3·28	925222	1·35	806971	4·63	193029	20
41	9·732390	3·28	9·925141	1·35	9·807249	4·63	10·192751	19
42	732587	3·28	925060	1·35	807527	4·63	192473	18
43	732784	3·28	924979	1·35	807805	4·63	192195	17
44	732980	3·27	924897	1·35	808083	4·63	191917	16
45	733177	3·27	924816	1·35	808361	4·63	191639	15
46	733373	3·27	924735	1·36	808638	4·62	191362	14
47	733569	3·27	924654	1·36	808916	4·62	191084	13
48	733765	3·27	924572	1·36	809193	4·62	190807	12
49	733961	3·26	924491	1·36	809471	4·62	190529	11
50	734157	3·26	924409	1·36	809748	4·62	190252	10
51	9·734353	3·26	9·924328	1·36	9·810025	4·62	10·189975	9
52	734549	3·26	924246	1·36	810302	4·62	189698	8
53	734744	3·25	924164	1·36	810580	4·62	189420	7
54	734939	3·25	924083	1·36	810857	4·62	189143	6
55	735135	3·25	924001	1·36	811134	4·61	188866	5
56	735330	3·25	923919	1·36	811410	4·61	188590	4
57	735525	3·25	923837	1·36	811687	4·61	188313	3
58	735719	3·24	923755	1·37	811964	4·61	188036	2
59	735914	3·24	923673	1·37	812241	4·61	187759	1
60	736109	3·24	923591	1·37	812517	4·61	187483	0
	Cosine	D.	Sine	57°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·736109	3·24	9·923591	1·37	9·812517	4·61	10·187483	60
1	736303	3·24	923509	1·37	812794	4·61	187206	59
2	736498	3·24	923427	1·37	813070	4·61	186930	58
3	736692	3·23	923345	1·37	813347	4·60	186653	57
4	736886	3·23	923263	1·37	813623	4·60	186377	56
5	737080	3·23	923181	1·37	813899	4·60	186101	55
6	737274	3·23	923098	1·37	814175	4·60	185825	54
7	737467	3·23	923016	1·37	814452	4·60	185548	53
8	737661	3·22	922933	1·37	814728	4·60	185272	52
9	737855	3·22	922851	1·37	815004	4·60	184996	51
10	738048	3·22	922768	1·38	815279	4·60	184721	50
11	9·738241	3·22	9·922686	1·38	9·815555	4·59	10·184445	49
12	738434	3·22	922603	1·38	815831	4·59	184169	48
13	738627	3·21	922520	1·38	816107	4·59	183893	47
14	738820	3·21	922438	1·38	816382	4·59	183618	46
15	739013	3·21	922355	1·38	816658	4·59	183342	45
16	739206	3·21	922272	1·38	816933	4·59	183067	44
17	739398	3·21	922189	1·38	817209	4·59	182791	43
18	739590	3·20	922106	1·38	817484	4·59	182516	42
19	739783	3·20	922023	1·38	817759	4·59	182241	41
20	739975	3·20	921940	1·38	818035	4·58	181965	40
21	9·740167	3·20	9·921857	1·39	9·818310	4·58	10·181690	39
22	740359	3·20	921774	1·39	818585	4·58	181415	38
23	740550	3·19	921691	1·39	818860	4·58	181140	37
24	740742	3·19	921607	1·39	819135	4·58	180865	36
25	740934	3·19	921524	1·39	819410	4·58	180590	35
26	741125	3·19	921441	1·39	819684	4·58	180316	34
27	741316	3·19	921357	1·39	819959	4·58	180041	33
28	741508	3·18	921274	1·39	820234	4·58	179766	32
29	741699	3·18	921190	1·39	820508	4·57	179492	31
30	741889	3·18	921107	1·39	820783	4·57	179217	30
31	9·742080	3·18	9·921023	1·39	9·821057	4·57	10·178943	29
32	742271	3·18	920939	1·40	821332	4·57	178668	28
33	742462	3·17	920856	1·40	821606	4·57	178394	27
34	742652	3·17	920772	1·40	821880	4·57	178120	26
35	742842	3·17	920688	1·40	822154	4·57	177846	25
36	743033	3·17	920604	1·40	822429	4·57	177571	24
37	743223	3·17	920520	1·40	822703	4·57	177297	23
38	743413	3·16	920436	1·40	822977	4·56	177023	22
39	743602	3·16	920352	1·40	823250	4·56	176750	21
40	743792	3·16	920268	1·40	823524	4·56	176476	20
41	9·743982	3·16	9·920184	1·40	9·823798	4·56	10·176202	19
42	744171	3·16	920099	1·40	824072	4·56	175928	18
43	744361	3·15	920015	1·40	824345	4·56	175655	17
44	744550	3·15	919931	1·41	824619	4·56	175381	16
45	744739	3·15	919846	1·41	824893	4·56	175107	15
46	744928	3·15	919762	1·41	825166	4·56	174834	14
47	745117	3·15	919677	1·41	825439	4·55	174561	13
48	745306	3·14	919593	1·41	825713	4·55	174287	12
49	745494	3·14	919508	1·41	825986	4·55	174014	11
50	745683	3·14	919424	1·41	826259	4·55	173741	10
51	9·745871	3·14	9·919339	1·41	9·826532	4·55	10·173468	9
52	746059	3·14	919254	1·41	826805	4·55	173195	8
53	746248	3·13	919169	1·41	827078	4·55	172922	7
54	746436	3·13	919085	1·41	827351	4·55	172649	6
55	746624	3·13	919000	1·41	827624	4·55	172376	5
56	746812	3·13	918915	1·42	827897	4·54	172103	4
57	746999	3·13	918830	1·42	828170	4·54	171830	3
58	747187	3·12	918745	1·42	828442	4·54	171558	2
59	747374	3·12	918659	1·42	828715	4·54	171285	1
60	747562	3·12	918574	1·42	828987	4·54	171013	0
	Cosine	D.	Sine	56°	Cotang.	D	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·747562	3·12	9·918574	1·42	9·828987	4·54	10·171013	60
1	747749	3·12	918489	1·42	829260	4·54	170740	59
2	747936	3·12	918404	1·42	829532	4·54	170468	58
3	748123	3·11	918318	1·42	829805	4·54	170195	57
4	748310	3·11	918233	1·42	830077	4·54	169923	56
5	748497	3·11	918147	1·42	830349	4·53	169651	55
6	748683	3·11	918062	1·42	830621	4·53	169379	54
7	748870	3·11	917976	1·43	830893	4·53	169107	53
8	749056	3·10	917891	1·43	831165	4·53	168835	52
9	749243	3·10	917805	1·43	831437	4·53	168563	51
10	749429	3·10	917719	1·43	831709	4·53	168291	50
11	9·749615	3·10	9·917634	1·43	9·831981	4·53	10·168019	49
12	749801	3·10	917548	1·43	832253	4·53	167747	48
13	749987	3·09	917462	1·43	832525	4·53	167475	47
14	750172	3·09	917376	1·43	832796	4·53	167204	46
15	750358	3·09	917290	1·43	833068	4·52	166932	45
16	750543	3·09	917204	1·43	833339	4·52	166661	44
17	750729	3·09	917118	1·44	833611	4·52	166389	43
18	750914	3·08	917032	1·44	833882	4·52	166118	42
19	751099	3·08	916946	1·44	834154	4·52	165846	41
20	751284	3·08	916859	1·44	834425	4·52	165575	40
21	9·751469	3·08	9·916773	1·44	9·834696	4·52	10·165304	39
22	751654	3·08	916687	1·44	834967	4·52	165033	38
23	751839	3·08	916600	1·44	835238	4·52	164762	37
24	752023	3·07	916514	1·44	835509	4·52	164491	36
25	752208	3·07	916427	1·44	835780	4·51	164220	35
26	752392	3·07	916341	1·44	836051	4·51	163949	34
27	752576	3·07	916254	1·44	836322	4·51	163678	33
28	752760	3·07	916167	1·45	836593	4·51	163407	32
29	752944	3·06	916081	1·45	836864	4·51	163136	31
30	753128	3·06	915994	1·45	837134	4·51	162866	30
31	9·753312	3·06	9·915907	1·45	9·837405	4·51	10·162595	29
32	753495	3·06	915820	1·45	837675	4·51	162325	28
33	753679	3·06	915733	1·45	837946	4·51	162054	27
34	753862	3·05	915646	1·45	838216	4·51	161784	26
35	754046	3·05	915559	1·45	838487	4·50	161513	25
36	754229	3·05	915472	1·45	838757	4·50	161243	24
37	754412	3·05	915385	1·45	839027	4·50	160973	23
38	754595	3·05	915297	1·45	839297	4·50	160703	22
39	754778	3·04	915210	1·45	839568	4·50	160432	21
40	754960	3·04	915123	1·46	839838	4·50	160162	20
41	9·755143	3·04	9·915035	1·46	9·840108	4·50	10·159892	19
42	755326	3·04	914948	1·46	840378	4·50	159622	18
43	755508	3·04	914860	1·46	840647	4·50	159353	17
44	755690	3·04	914773	1·46	840917	4·49	159083	16
45	755872	3·03	914685	1·46	841187	4·49	158813	15
46	756054	3·03	914598	1·46	841457	4·49	158543	14
47	756236	3·03	914510	1·46	841726	4·49	158274	13
48	756418	3·03	914422	1·46	841996	4·49	158004	12
49	756600	3·03	914334	1·46	842266	4·49	157734	11
50	756782	3·02	914246	1·47	842535	4·49	157465	10
51	9·756963	3·02	9·914158	1·47	9·842805	4·49	10·157195	9
52	757144	3·02	914070	1·47	843074	4·49	156926	8
53	757326	3·02	913982	1·47	843343	4·49	156657	7
54	757507	3·02	913894	1·47	843612	4·49	156388	6
55	757688	3·01	913806	1·47	843882	4·48	156118	5
56	757869	3·01	913718	1·47	844151	4·48	155849	4
57	758050	3·01	913630	1·47	844420	4·48	155580	3
58	758230	3·01	913541	1·47	844689	4·48	155311	2
59	758411	3·01	913453	1·47	844958	4·48	155042	1
60	758591	3·01	913365	1·47	845227	4·48	154773	0
	Cosine	D.	Sine	55°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·758591	3·01	9·913365	1·47	9·845227	4·48	10·154773	60
1	758772	3·00	913276	1·47	845496	4·48	154504	59
2	758952	3·00	913187	1·48	845764	4·48	154236	58
3	759132	3·00	913099	1·48	846033	4·48	153967	57
4	759312	3·00	913010	1·48	846302	4·48	153698	56
5	759492	3·00	912922	1·48	846570	4·47	153430	55
6	759672	2·99	912833	1·48	846839	4·47	153161	54
7	759852	2·99	912744	1·48	847107	4·47	152893	53
8	760031	2·99	912655	1·48	847376	4·47	152624	52
9	760211	2·99	912566	1·48	847644	4·47	152356	51
10	760390	2·99	912477	1·48	847913	4·47	152087	50
11	9·760569	2·98	9·912388	1·48	9·848181	4·47	10·151819	49
12	760748	2·98	912299	1·49	848449	4·47	151551	48
13	760927	2·98	912210	1·49	848717	4·47	151283	47
14	761106	2·98	912121	1·49	848986	4·47	151014	46
15	761285	2·98	912031	1·49	849254	4·47	150746	45
16	761464	2·98	911942	1·49	849522	4·47	150478	44
17	761642	2·97	911853	1·49	849790	4·46	150210	43
18	761821	2·97	911763	1·49	850058	4·46	149942	42
19	761999	2·97	911674	1·49	850325	4·46	149675	41
20	762177	2·97	911584	1·49	850593	4·46	149407	40
21	9·762356	2·97	9·911495	1·49	9·850861	4·46	10·149139	39
22	762534	2·96	911405	1·49	851129	4·46	148871	38
23	762712	2·96	911315	1·50	851396	4·46	148604	37
24	762889	2·96	911226	1·50	851664	4·46	148336	36
25	763067	2·96	911136	1·50	851931	4·46	148069	35
26	763245	2·96	911046	1·50	852199	4·46	147801	34
27	763422	2·96	910956	1·50	852466	4·46	147534	33
28	763600	2·95	910866	1·50	852733	4·45	147267	32
29	763777	2·95	910776	1·50	853001	4·45	146999	31
30	763954	2·95	910686	1·50	853268	4·45	146732	30
31	9·764131	2·95	9·910596	1·50	9·853535	4·45	10·146465	29
32	764308	2·95	910506	1·50	853802	4·45	146198	28
33	764485	2·94	910415	1·50	854069	4·45	145931	27
34	764662	2·94	910325	1·51	854336	4·45	145664	26
35	764838	2·94	910235	1·51	854603	4·45	145397	25
36	765015	2·94	910144	1·51	854870	4·45	145130	24
37	765191	2·94	910054	1·51	855137	4·45	144863	23
38	765367	2·94	909963	1·51	855404	4·45	144596	22
39	765544	2·93	909873	1·51	855671	4·44	144329	21
40	765720	2·93	909782	1·51	855938	4·44	144062	20
41	9·765896	2·93	9·909691	1·51	9·856204	4·44	10·143796	19
42	766072	2·93	909601	1·51	856471	4·44	143529	18
43	766247	2·93	909510	1·51	856737	4·44	143263	17
44	766423	2·93	909419	1·51	857004	4·44	142996	16
45	766598	2·92	909328	1·52	857270	4·44	142730	15
46	766774	2·92	909237	1·52	857537	4·44	142463	14
47	766949	2·92	909146	1·52	857803	4·44	142197	13
48	767124	2·92	909055	1·52	858069	4·44	141931	12
49	767300	2·92	908964	1·52	858336	4·44	141664	11
50	767475	2·91	908873	1·52	858602	4·43	141398	10
51	9·767649	2·91	9·908781	1·52	9·858868	4·43	10·141132	9
52	767824	2·91	908690	1·52	859134	4·43	140866	8
53	767999	2·91	908599	1·52	859400	4·43	140600	7
54	768173	2·91	908507	1·52	859666	4·43	140334	6
55	768348	2·90	908416	1·53	859932	4·43	140068	5
56	768522	2·90	908324	1·53	860198	4·43	139802	4
57	768697	2·90	908233	1·53	860464	4·43	139536	3
58	768871	2·90	908141	1·53	860730	4·43	139270	2
59	769045	2·90	908049	1·53	860995	4·43	139005	1
60	769219	2·90	907958	1·53	861261	4·43	138739	0
	Cosine	D.	Sine	54°	Cotang	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·769219	2·90	9·907958	1·53	9·861261	4·43	10·138739	60
1	769393	2·89	907866	1·53	861527	4·43	138473	59
2	769566	2·89	907774	1·53	861792	4·42	138208	58
3	769740	2·89	907682	1·53	862058	4·42	137942	57
4	769913	2·89	907590	1·53	862323	4·42	137677	56
5	770087	2·89	907498	1·53	862589	4·42	137411	55
6	770260	2·88	907406	1·53	862854	4·42	137146	54
7	770433	2·88	907314	1·54	863119	4·42	136881	53
8	770606	2·88	907222	1·54	863385	4·42	136615	52
9	770779	2·88	907129	1·54	863650	4·42	136350	51
10	770952	2·88	907037	1·54	863915	4·42	136085	50
11	9·771125	2·88	9·906945	1·54	9·864180	4·42	10·135820	49
12	771298	2·87	906852	1·54	864445	4·42	135555	48
13	771470	2·87	906760	1·54	864710	4·42	135290	47
14	771643	2·87	906667	1·54	864975	4·41	135025	46
15	771815	2·87	906575	1·54	865240	4·41	134760	45
16	771987	2·87	906482	1·54	865505	4·41	134495	44
17	772159	2·87	906389	1·55	865770	4·41	134230	43
18	772331	2·86	906296	1·55	866035	4·41	133965	42
19	772503	2·86	906204	1·55	866300	4·41	133700	41
20	772675	2·86	906111	1·55	866564	4·41	133436	40
21	9·772847	2·86	9·906018	1·55	9·866829	4·41	10·133171	39
22	773018	2·86	905925	1·55	867094	4·41	132906	38
23	773190	2·86	905832	1·55	867358	4·41	132642	37
24	773361	2·85	905739	1·55	867623	4·41	132377	36
25	773533	2·85	905645	1·55	867887	4·41	132113	35
26	773704	2·85	905552	1·55	868152	4·40	131848	34
27	773875	2·85	905459	1·55	868416	4·40	131584	33
28	774046	2·85	905366	1·56	868680	4·40	131320	32
29	774217	2·85	905272	1·56	868945	4·40	131055	31
30	774388	2·84	905179	1·56	869209	4·40	130794	30
31	9·774558	2·84	9·905085	1·56	9·869473	4·40	10·130527	29
32	774729	2·84	904992	1·56	869737	4·40	130263	28
33	774899	2·84	904898	1·56	870001	4·40	129999	27
34	775070	2·84	904804	1·56	870265	4·40	129735	26
35	775240	2·84	904711	1·56	870529	4·40	129471	25
36	775410	2·83	904617	1·56	870793	4·40	129207	24
37	775580	2·83	904523	1·56	871057	4·40	128943	23
38	775750	2·83	904429	1·57	871321	4·40	128679	22
39	775920	2·83	904335	1·57	871585	4·40	128415	21
40	776090	2·83	904241	1·57	871849	4·39	128151	20
41	9·776259	2·83	9·904147	1·57	9·872112	4·39	10·127888	19
42	776429	2·82	904053	1·57	872376	4·39	127624	18
43	776598	2·82	903959	1·57	872640	4·39	127360	17
44	776768	2·82	903864	1·57	872903	4·39	127097	16
45	776937	2·82	903770	1·57	873167	4·39	126833	15
46	777106	2·82	903676	1·57	873430	4·39	126570	14
47	777275	2·81	903581	1·57	873694	4·39	126306	13
48	777444	2·81	903487	1·57	873957	4·39	126043	12
49	777613	2·81	903392	1·58	874220	4·39	125780	11
50	777781	2·81	903298	1·58	874484	4·39	125516	10
51	9·777950	2·81	9·903203	1·58	9·874747	4·39	10 125253	9
52	778119	2·81	903108	1·58	875010	4·39	124990	8
53	778287	2·80	903014	1·58	875273	4·38	124727	7
54	778455	2·80	902919	1·58	875536	4·38	124464	6
55	778624	2·80	902824	1·58	875800	4·38	124200	5
56	778792	2·80	902729	1·58	876063	4·38	123937	4
57	778960	2·80	902634	1·58	876326	4·38	123674	3
58	779128	2·80	902539	1·59	876589	4·38	123411	2
59	779295	2·79	902444	1·59	876851	4·38	123149	1
60	779463	2·79	902349	1·59	877114	4·38	122886	0
	Cosine	D.	Sine	53°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·779463	2·79	9·902349	1·59	9·877114	4·38	10·122886	60
1	779631	2·79	902253	1·59	877377	4·38	122623	59
2	779798	2·79	902158	1·59	877640	4·38	122360	58
3	779966	2·79	902063	1·59	877903	4·38	122097	57
4	780133	2·79	901967	1·59	878165	4·38	121835	56
5	780300	2·78	901872	1·59	878428	4·38	121572	55
6	780467	2·78	901776	1·59	878691	4·38	121309	54
7	780634	2·78	901681	1·59	878953	4·37	121047	53
8	780801	2·78	901585	1·59	879216	4·37	120784	52
9	780968	2·78	901490	1·59	879478	4·37	120522	51
10	781134	2·78	901394	1·60	879741	4·37	120259	50
11	9·781301	2·77	9·901298	1·60	9·880003	4·37	10·119997	49
12	781468	2·77	901202	1·60	880265	4·37	119735	48
13	781634	2·77	901106	1·60	880528	4·37	119472	47
14	781800	2·77	901010	1·60	880790	4·37	119210	46
15	781966	2·77	900914	1·60	881052	4·37	118948	45
16	782132	2·77	900818	1·60	881314	4·37	118686	44
17	782298	2·76	900722	1·60	881576	4·37	118424	43
18	782464	2·76	900626	1·60	881839	4·37	118161	42
19	782630	2·76	900529	1·60	882101	4·37	117899	41
20	782796	2·76	900433	1·61	882363	4·36	117637	40
21	9·782961	2·76	9·900337	1·61	9·882625	4·36	10·117375	39
22	783127	2·76	900240	1·61	882887	4·36	117113	38
23	783292	2·75	900144	1·61	883148	4·36	116852	37
24	783458	2·75	900047	1·61	883410	4·36	116590	36
25	783623	2·75	899951	1·61	883672	4·36	116328	35
26	783788	2·75	899854	1·61	883934	4·36	116066	34
27	783953	2·75	899757	1·61	884196	4·36	115804	33
28	784118	2·75	899660	1·61	884457	4·36	115543	32
29	784282	2·74	899564	1·61	884719	4·36	115281	31
30	784447	2·74	899467	1·62	884980	4·36	115020	30
31	9·784612	2·74	9·899370	1·62	9·885242	4·36	10·114758	29
32	784776	2·74	899273	1·62	885503	4·36	114497	28
33	784941	2·74	899176	1·62	885765	4·36	114235	27
34	785105	2·74	899078	1·62	886026	4·36	113974	26
35	785269	2·73	898981	1·62	886288	4·36	113712	25
36	785433	2·73	898884	1·62	886549	4·35	113451	24
37	785597	2·73	898787	1·62	886810	4·35	113190	23
38	785761	2·73	898689	1·62	887072	4·35	112928	22
39	785925	2·73	898592	1·62	887333	4·35	112667	21
40	786089	2·73	898494	1·63	887594	4·35	112406	20
41	9·786252	2·72	9·898397	1·63	9·887855	4·35	10·112145	19
42	786416	2·72	898299	1·63	888116	4·35	111884	18
43	786579	2·72	898202	1·63	888377	4·35	111623	17
44	786742	2·72	898104	1·63	888639	4·35	111361	16
45	786906	2·72	898006	1·63	888900	4·35	111100	15
46	787069	2·72	897908	1·63	889160	4·35	110840	14
47	787232	2·71	897810	1·63	889421	4·35	110579	13
48	787395	2·71	897712	1·63	889682	4·35	110318	12
49	787557	2·71	897614	1·63	889943	4·35	110057	11
50	787720	2·71	897516	1·63	890204	4·34	109796	10
51	9·787883	2·71	9·897418	1·64	9·890465	4·34	10·109535	9
52	788045	2·71	897320	1·64	890725	4·34	109275	8
53	788208	2·71	897222	1·64	890986	4·34	109014	7
54	788370	2·70	897123	1·64	891247	4·34	108753	6
55	788532	2·70	897025	1·64	891507	4·34	108493	5
56	788694	2·70	896926	1·64	891768	4·34	108232	4
57	788856	2·70	896828	1·64	892028	4·34	107972	3
58	789018	2·70	896729	1·64	892289	4·34	107711	2
59	789180	2·70	896631	1·64	892549	4·34	107451	1
60	789342	2·69	896532	1·64	892810	4·34	107190	0
	Cosine	D.	Sine	52°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·789342	2·69	9·896532	1·64	9·892810	4·34	10·107190	60
1	789504	2·69	896433	1·65	893070	4·34	106930	59
2	789665	2·69	896335	1·65	893331	4·34	106669	58
3	789827	2·69	896236	1·65	893591	4·34	106409	57
4	789988	2·69	896137	1·65	893851	4·34	106149	56
5	790149	2·69	896038	1·65	894111	4·34	105889	55
6	790310	2·68	895939	1·65	894371	4·34	105629	54
7	790471	2·68	895840	1·65	894632	4·33	105368	53
8	790632	2·68	895741	1·65	894892	4·33	105108	52
9	790793	2·68	895641	1·65	895152	4·33	104848	51
10	790954	2·68	895542	1·65	895412	4·33	104588	50
11	9·791115	2·68	9·895443	1·66	9·895672	4·33	10·104328	49
12	791275	2·67	895343	1·66	895932	4·33	104068	48
13	791436	2·67	895244	1·66	896192	4·33	103808	47
14	791596	2·67	895145	1·66	896452	4·33	103548	46
15	791757	2·67	895045	1·66	896712	4·33	103288	45
16	791917	2·67	894945	1·66	896971	4·33	103029	44
17	792077	2·67	894846	1·66	897231	4·33	102769	43
18	792237	2·66	894746	1·66	897491	4·33	102509	42
19	792397	2·66	894646	1·66	897751	4·33	102249	41
20	792557	2·66	894546	1·66	898010	4·33	101990	40
21	9·792716	2·66	9·894446	1·67	9·898270	4·33	10·101730	39
22	792876	2·66	894346	1·67	898530	4·33	101470	38
23	793035	2·66	894246	1·67	898789	4·33	101211	37
24	793195	2·65	894146	1·67	899049	4·32	100951	36
25	793354	2·65	894046	1·67	899308	4·32	100692	35
26	793514	2·65	893946	1·67	899568	4·32	100432	34
27	793673	2·65	893846	1·67	899827	4·32	100173	33
28	793832	2·65	893745	1·67	900086	4·32	099914	32
29	793991	2·65	893645	1·67	900346	4·32	099654	31
30	794150	2·64	893544	1·67	900605	4·32	099395	30
31	9·794308	2·64	9·893444	1·68	9·900864	4·32	10·099136	29
32	794467	2·64	893343	1·68	901124	4·32	098876	28
33	794626	2·64	893243	1·68	901383	4·32	098617	27
34	794784	2·64	893142	1·68	901642	4·32	098358	26
35	794942	2·64	893041	1·68	901901	4·32	098099	25
36	795101	2·64	892940	1·68	902160	4·32	097840	24
37	795259	2·63	892839	1·68	902419	4·32	097581	23
38	795417	2·63	892739	1·68	902679	4·32	097321	22
39	795575	2·63	892638	1·68	902938	4·32	097062	21
40	795733	2·63	892536	1·68	903197	4·31	096803	20
41	9·795891	2·63	9·892435	1·69	9·903455	4·31	10·096545	19
42	796049	2·63	892334	1·69	903714	4·31	096286	18
43	796206	2·63	892233	1·69	903973	4·31	096027	17
44	796364	2·62	892132	1·69	904232	4·31	095768	16
45	796521	2·62	892030	1·69	904491	4·31	095509	15
46	796679	2·62	891929	1·69	904750	4·31	095250	14
47	796836	2·62	891827	1·69	905008	4·31	094992	13
48	796993	2·62	891726	1·69	905267	4·31	094733	12
49	797150	2·61	891624	1·69	905526	4·31	094474	11
50	797307	2·61	891523	1·70	905784	4·31	094216	10
51	9·797464	2·61	9·891421	1·70	9·906043	4·31	10·093957	9
52	797621	2·61	891319	1·70	906302	4·31	093698	8
53	797777	2·61	891217	1·70	906560	4·31	093440	7
54	797934	2·61	891115	1·70	906819	4·31	093181	6
55	798091	2·61	891013	1·70	907077	4·31	092923	5
56	798247	2·61	890911	1·70	907336	4·31	092664	4
57	798403	2·60	890809	1·70	907594	4·31	092406	3
58	798560	2·60	890707	1·70	907852	4·31	092148	2
59	798716	2·60	890605	1·70	908111	4·30	091889	1
60	798872	2·60	890503	1·70	908369	4·30	091631	0
	Cosine	D.	Sine	51°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·798872	2·60	9·890503	1·70	9·908369	4·30	10·091631	60
1	799028	2·60	890400	1·71	908628	4·30	091372	59
2	799184	2·60	890298	1·71	908886	4·30	091114	58
3	799339	2·59	890195	1·71	909144	4·30	090856	57
4	799495	2·59	890093	1·71	909402	4·30	090598	56
5	799651	2·59	889990	1·71	909660	4·30	090340	55
6	799806	2·59	889888	1·71	909918	4·30	090082	54
7	799962	2·59	889785	1·71	910177	4·30	089823	53
8	800117	2·59	889682	1·71	910435	4·30	089565	52
9	800272	2·58	889579	1·71	910693	4·30	089307	51
10	800427	2·58	889477	1·71	910951	4·30	089049	50
11	9·800582	2·58	9·889374	1·72	9·911209	4·30	10·088791	49
12	800737	2·58	889271	1·72	911467	4·30	088533	48
13	800892	2·58	889168	1·72	911724	4·30	088276	47
14	801047	2·58	889064	1·72	911982	4·30	088018	46
15	801201	2·58	888961	1·72	912240	4·30	087760	45
16	801356	2·57	888858	1·72	912498	4·30	087502	44
17	801511	2·57	888755	1·72	912756	4·30	087244	43
18	801665	2·57	888651	1·72	913014	4·29	086986	42
19	801819	2·57	888548	1·72	913271	4·29	086729	41
20	801973	2·57	888444	1·73	913529	4·29	086471	40
21	9·802128	2·57	9·888341	1·73	9·913787	4·29	10·086213	39
22	802282	2·56	888237	1·73	914044	4·29	085956	38
23	802436	2·56	888134	1·73	914302	4·29	085698	37
24	802589	2·56	888030	1·73	914560	4·29	085440	36
25	802743	2·56	887926	1·73	914817	4·29	085183	35
26	802897	2·56	887822	1·73	915075	4·29	084925	34
27	803050	2·56	887718	1·73	915332	4·29	084668	33
28	803204	2·56	887614	1·73	915590	4·29	084410	32
29	803357	2·55	887510	1·73	915847	4·29	084153	31
30	803511	2·55	887406	1·74	916104	4·29	083896	30
31	9·803664	2·55	9·887302	1·74	9·916362	4·29	10·083638	29
32	803817	2·55	887198	1·74	916619	4·29	083381	28
33	803970	2·55	887093	1·74	916877	4·29	083123	27
34	804123	2·55	886989	1·74	917134	4·29	082866	26
35	804276	2·54	886885	1·74	917391	4·29	082609	25
36	804428	2·54	886780	1·74	917648	4·29	082352	24
37	804581	2·54	886676	1·74	917905	4·29	082095	23
38	804734	2·54	886571	1·74	918163	4·28	081837	22
39	804886	2·54	886466	1·74	918420	4·28	081580	21
40	805039	2·54	886362	1·75	918677	4·28	081323	20
41	9·805191	2·54	9·886257	1·75	9·918934	4·28	10·081066	19
42	805343	2·53	886152	1·75	919191	4·28	080809	18
43	805495	2·53	886047	1·75	919448	4·28	080552	17
44	805647	2·53	885942	1·75	919705	4·28	080295	16
45	805799	2·53	885837	1·75	919962	4·28	080038	15
46	805951	2·53	885732	1·75	920219	4·28	079781	14
47	806103	2·53	885627	1·75	920476	4·28	079524	13
48	806254	2·53	885522	1·75	920733	4·28	079267	12
49	806406	2·52	885416	1·75	920990	4·28	079010	11
50	806557	2·52	885311	1·76	921247	4·28	078753	10
51	9·806709	2·52	9·885205	1·76	9·921503	4·28	10·078497	9
52	806860	2·52	885100	1·76	921760	4·28	078240	8
53	807011	2·52	884994	1·76	922017	4·28	077983	7
54	807163	2·52	884889	1·76	922274	4·28	077726	6
55	807314	2·52	884783	1·76	922530	4·28	077470	5
56	807465	2·51	884677	1·76	922787	4·28	077213	4
57	807615	2·51	884572	1·76	923044	4·28	076956	3
58	807766	2·51	884466	1·76	923300	4·28	076700	2
59	807917	2·51	884360	1·76	923557	4·27	076443	1
60	808067	2·51	884254	1·77	923813	4·27	076187	0
	Cosine	D.	Sine	50°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·808067	2·51	9·884254	1·77	9·923813	4·27	10·076187	60
1	808218	2·51	884148	1·77	924070	4·27	075930	59
2	808368	2·51	884042	1·77	924327	4·27	075673	58
3	808519	2·50	883936	1·77	924583	4·27	075417	57
4	808669	2·50	883829	1·77	924840	4·27	075160	56
5	808819	2·50	883723	1·77	925096	4·27	074904	55
6	808969	2·50	883617	1·77	925352	4·27	074648	54
7	809119	2·50	883510	1·77	925609	4·27	074391	53
8	809269	2·50	883404	1·77	925865	4·27	074135	52
9	809419	2·49	883297	1·78	926122	4·27	073878	51
10	809569	2·49	883191	1·78	926378	4·27	073622	50
11	9·809718	2·49	9·883084	1·78	9·926634	4·27	10·073366	49
12	809868	2·49	882977	1·78	926890	4·27	073110	48
13	810017	2·49	882871	1·78	927147	4·27	072853	47
14	810167	2·49	882764	1·78	927403	4·27	072597	46
15	810316	2·48	882657	1·78	927659	4·27	072341	45
16	810465	2·48	882550	1·78	927915	4·27	072085	44
17	810614	2·48	882443	1·78	928171	4·27	071829	43
18	810763	2·48	882336	1·79	928427	4·27	071573	42
19	810912	2·48	882229	1·79	928683	4·27	071317	41
20	811061	2·48	882121	1·79	928940	4·27	071060	40
21	9·811210	2·48	9·882014	1·79	9·929196	4·27	10·070804	39
22	811358	2·47	881907	1·79	929452	4·27	070548	38
23	811507	2·47	881799	1·79	929708	4·27	070292	37
24	811655	2·47	881692	1·79	929964	4·26	070036	36
25	811804	2·47	881584	1·79	930220	4·26	069780	35
26	811952	2·47	881477	1·79	930475	4·26	069525	34
27	812100	2·47	881369	1·79	930731	4·26	069269	33
28	812248	2·47	881261	1·80	930987	4·26	069013	32
29	812396	2·46	881153	1·80	931243	4·26	068757	31
30	812544	2·46	881046	1·80	931499	4·26	068501	30
31	9·812692	2·46	9·880938	1·80	9·931755	4·26	10·068245	29
32	812840	2·46	880830	1·80	932010	4·26	067990	28
33	812988	2·46	880722	1·80	932266	4·26	067734	27
34	813135	2·46	880613	1·80	932522	4·26	067478	26
35	813283	2·46	880505	1·80	932778	4·26	067222	25
36	813430	2·45	880397	1·80	933033	4·26	066967	24
37	813578	2·45	880289	1·81	933289	4·26	066711	23
38	813725	2·45	880180	1·81	933545	4·26	066455	22
39	813872	2·45	880072	1·81	933800	4·26	066200	21
40	814019	2·45	879963	1·81	934056	4·26	065944	20
41	9·814166	2·45	9·879855	1·81	9·934311	4·26	10·065689	19
42	814313	2·45	879746	1·81	934567	4·26	065433	18
43	814460	2·44	879637	1·81	934823	4·26	065177	17
44	814607	2·44	879529	1·81	935078	4·26	064922	16
45	814753	2·44	879420	1·81	935333	4·26	064667	15
46	814900	2·44	879311	1·81	935589	4·26	064411	14
47	815046	2·44	879202	1·82	935844	4·26	064156	13
48	815193	2·44	879093	1·82	936100	4·26	063900	12
49	815339	2·44	878984	1·82	936355	4·26	063645	11
50	815485	2·43	878875	1·82	936610	4·26	063390	10
51	9·815631	2·43	9·878766	1·82	9·936866	4·25	10·063134	9
52	815778	2·43	878656	1·82	937121	4·25	062879	8
53	815924	2·43	878547	1·82	937376	4·25	062624	7
54	816069	2·43	878438	1·82	937632	4·25	062368	6
55	816215	2·43	878328	1·82	937887	4·25	062113	5
56	816361	2·43	878219	1·83	938142	4·25	061858	4
57	816507	2·42	878109	1·83	938398	4·25	061602	3
58	816652	2·42	877999	1·83	938653	4·25	061347	2
59	816798	2·42	877890	1·83	938908	4·25	061092	1
60	816943	2·42	877780	1·83	939163	4·25	060837	0
	Cosine	D.	Sine	49°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·816943	2·42	9·877780	1·83	9·939163	4·25	10·060837	60
1	817088	2·42	877670	1·83	939418	4·25	060582	59
2	817233	2·42	877560	1·83	939673	4·25	060327	58
3	817379	2·42	877450	1·83	939928	4·25	060072	57
4	817524	2·41	877340	1 83	940183	4·25	059817	56
5	817668	2·41	877230	1·84	940438	4·25	059562	55
6	817813	2·41	877120	1 84	940694	4·25	059306	54
7	817958	2·41	877010	1·84	940949	4·25	059051	53
8	818103	2·41	876899	1·84	94[illegible]204	4·25	058796	52
9	818247	2·41	876789	1·84	94[illegible]458	4·25	058542	51
10	818392	2·41	876678	1·84	94[illegible]14	4·25	058286	50
11	9·818536	2·40	9·876568	1·84	9·041968	4·25	10·058032	49
12	818681	2·40	876457	1·84	94[illegible]2[illegible]3	4·25	057777	48
13	818825	2·40	876347	1·84	942478	4·25	057522	47
14	818969	2·40	876236	1·85	94[illegible]733	4·25	057267	46
15	819113	2·40	876125	1·85	9[illegible]88	4·25	057012	45
16	819257	2·40	876014	1·85	94[illegible]243	4·25	056757	44
17	819401	2·40	875904	1·85	943498	4·25	056502	43
18	819545	2·39	875793	1·85	943752	4·25	056248	42
19	819689	2·39	875682	1·85	944007	4·25	055993	41
20	819832	2·39	875571	1·85	944262	4·25	055738	40
21	9·819976	2·39	9·875459	1·85	9·944517	4·25	10·055483	39
22	820120	2·39	875348	1·85	944771	4·24	055229	38
23	820263	2·39	875237	1·85	945026	4·24	054974	37
24	820406	2·39	875126	1·86	945281	4·24	054719	36
25	820550	2·38	875014	1·86	945535	4·24	054465	35
26	820693	2·38	874903	1·86	945790	4·24	054210	34
27	820836	2·38	874791	1·86	946045	4·24	053955	33
28	820979	2·38	874680	1·86	946299	4·24	053701	32
29	821122	2·38	874568	1·86	946554	4·24	053446	31
30	821265	2·38	874456	1·86	946808	4·24	053192	30
31	9·821407	2·38	9·874344	1·86	9·947063	4·24	10·052937	29
32	821550	2·38	874232	1·87	947318	4·24	052682	28
33	821693	2·37	874121	1·87	947572	4·24	052428	27
34	821835	2·37	874009	1·87	947826	4·24	052174	26
35	821977	2·37	873896	1·87	948081	4·24	051919	25
36	822120	2·37	873784	1·87	948336	4·24	051664	24
37	822262	2·37	873672	1·87	948590	4·24	051410	23
38	822404	2·37	873560	1·87	948844	4·24	051156	22
39	822546	2·37	873448	1·87	949099	4·24	050901	21
40	822688	2·36	873335	1·87	949353	4·24	050647	20
41	9·822830	2·36	9·873223	1·87	9·949607	4·24	10·050393	19
42	822972	2·36	873110	1·88	949862	4·24	050138	18
43	823114	2·36	872998	1·88	950116	4·24	049884	17
44	823255	2·36	872885	1·88	950370	4·24	049630	16
45	823397	2·36	872772	1·88	950625	4·24	049375	15
46	823539	2·36	872659	1·88	950879	4·24	049121	14
47	823680	2·35	872547	1·88	951133	4·24	048867	13
48	823821	2·35	872434	1·88	951388	4·24	048612	12
49	823963	2·35	872321	1·88	951642	4·24	048358	11
50	824104	2·35	872208	1·88	951896	4·24	048104	10
51	9·824245	2·35	9·872095	1·89	9·952150	4·24	10·047850	9
52	824386	2·35	871981	1·89	952405	4·24	047595	8
53	824527	2·35	871868	1·89	952659	4 24	047341	7
54	824668	2·34	871755	1·89	952913	4·24	047087	6
55	824808	2·34	871641	1·89	953167	4·23	046833	5
56	824949	2·34	871528	1·89	953421	4·23	046579	4
57	825090	2·34	871414	1·89	953675	4·23	046325	3
58	825230	2·34	871301	1·89	953929	4·23	046071	2
59	825371	2·34	871187	1·89	954183	4·23	045817	1
60	825511	2·34	871073	1·90	954437	4·23	045563	0
	Cosine	D.	Sine	48°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·825511	2·34	9·871073	1·90	9·954437	4·23	10·045563	60
1	825651	2·33	870960	1·90	954691	4·23	045309	59
2	825791	2·33	870846	1·90	954945	4·23	045055	58
3	825931	2·33	870732	1·90	955200	4·23	044800	57
4	826071	2·33	870618	1·90	955454	4·23	044546	56
5	826211	2·33	870504	1·90	955707	4·23	044293	55
6	826351	2·33	870390	1·90	955961	4·23	044039	54
7	826491	2·33	870276	1·90	956215	4·23	043785	53
8	826631	2·33	870161	1·90	956469	4·23	043531	52
9	826770	2·32	870047	1·91	956723	4·23	043277	51
10	826910	2·32	869933	1·91	956977	4·23	043023	50
11	9·827049	2·32	9·869818	1·91	9·957231	4·23	10·042769	49
12	827189	2·32	869704	1·91	957485	4·23	042515	48
13	827328	2·32	869589	1·91	957739	4·23	042261	47
14	827467	2·32	869474	1·91	957993	4·23	042007	46
15	827606	2·32	869360	1·91	958246	4·23	041754	45
16	827745	2·32	869245	1·91	958500	4·23	041500	44
17	827884	2·31	869130	1·91	958754	4·23	041246	43
18	828023	2·31	869015	1·92	959008	4·23	040992	42
19	828162	2·31	868900	1·92	959262	4·23	040738	41
20	828301	2·31	868785	1·92	959516	4·23	040484	40
21	9·828439	2·31	9·868670	1·92	9·959769	4·23	10·040231	39
22	828578	2·31	868555	1·92	960023	4·23	039977	38
23	828716	2·31	868440	1·92	960277	4·23	039723	37
24	828855	2·30	868324	1·92	960531	4·23	039469	36
25	828993	2·30	868209	1·92	960784	4·23	039216	35
26	829131	2·30	868093	1·92	961038	4·23	038962	34
27	829269	2·30	867978	1·93	961291	4·23	038709	33
28	829407	2·30	867862	1·93	961545	4·23	038455	32
29	829545	2·30	867747	1·93	961799	4·23	038201	31
30	829683	2·30	867631	1·93	962052	4·23	037948	30
31	9·829821	2·29	9·867515	1·93	9·962306	4·23	10·037694	29
32	829959	2·29	867399	1·93	962560	4·23	037440	28
33	830097	2·29	867283	1·93	962813	4·23	037187	27
34	830234	2·29	867167	1·93	963067	4·23	036933	26
35	830372	2·29	867051	1·93	963320	4·23	036680	25
36	830509	2·29	866935	1·94	963574	4·23	036426	24
37	830646	2·29	866819	1·94	963827	4·23	036173	23
38	830784	2·29	866703	1·94	964081	4·23	035919	22
39	830921	2·28	866586	1·94	964335	4·23	035665	21
40	831058	2·28	866470	1·94	964588	4·22	035412	20
41	9·831195	2·28	9·866353	1·94	9·964842	4·22	10·035158	19
42	831332	2·28	866237	1·94	965095	4·22	034905	18
43	831469	2·28	866120	1·94	965349	4·22	034651	17
44	831606	2·28	866004	1·95	965602	4·22	034398	16
45	831742	2·28	865887	1·95	965855	4·22	034145	15
46	831879	2·28	865770	1·95	966105	4·22	033891	14
47	832015	2·27	865653	1·95	966362	4·22	033638	13
48	832152	2·27	865536	1·95	966616	4·22	033384	12
49	832288	2·27	865419	1·95	966869	4·22	033131	11
50	832425	2·27	865302	1·95	967123	4·22	032877	10
51	9·832561	2·27	9·865185	1·95	9·967376	4·22	10·032624	9
52	832697	2·27	865068	1·95	967629	4·22	032371	8
53	832833	2·27	864950	1·95	967883	4·22	032117	7
54	832969	2·26	864833	1·96	968136	4·22	031864	6
55	833105	2·26	864716	1·96	968389	4·22	031611	5
56	833241	2·26	864598	1·96	968643	4·22	031357	4
57	833377	2·26	864481	1·96	968896	4·22	031104	3
58	833512	2·26	864363	1·96	969149	4·22	030851	2
59	833648	2·26	864245	1·96	969403	4·22	030597	1
60	833783	2·26	864127	1·96	969656	4·22	030344	0
	Cosine	D.	Sine	47°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·833783	2·26	9·864127	1·96	9·969656	4·22	10·030344	60
1	833919	2·25	864010	1·96	969909	4·22	030091	59
2	834054	2·25	863892	1·97	970162	4·22	029838	58
3	834189	2·25	863774	1·97	970416	4·22	029584	57
4	834325	2·25	863656	1·97	970669	4·22	029331	56
5	834460	2·25	863538	1·97	970922	4·22	029078	55
6	834595	2·25	863419	1·97	971175	4·22	028825	54
7	834730	2·25	863301	1·97	971429	4·22	028571	53
8	834865	2·25	863183	1·97	971682	4·22	028318	52
9	834999	2·24	863064	1·97	971935	4·22	028065	51
10	835134	2·24	862946	1·98	972188	4·22	027812	50
11	9·835269	2·24	9·862827	1·98	9·972441	4·22	10·027559	49
12	835403	2·24	862709	1·98	972694	4·22	027306	48
13	835538	2·24	862590	1·98	972948	4·22	027052	47
14	835672	2·24	862471	1·98	973201	4·22	026799	46
15	835807	2·24	862353	1·98	973454	4·22	026546	45
16	835941	2·24	862234	1·98	973707	4·22	026293	44
17	836075	2·23	862115	1·98	973960	4·22	026040	43
18	836209	2·23	861996	1·98	974213	4·22	025787	42
19	836343	2·23	861877	1·98	974466	4·22	025534	41
20	836477	2·23	861758	1·99	974719	4·22	025281	40
21	9·836611	2·23	9·861638	1·99	9·974973	4·22	10·025027	39
22	836745	2·23	861519	1·99	975226	4·22	024774	38
23	836878	2·23	861400	1·99	975479	4·22	024521	37
24	837012	2·22	861280	1·99	975732	4·22	024268	36
25	837146	2·22	861161	1·99	975985	4·22	024015	35
26	837279	2·22	861041	1·99	976238	4·22	023762	34
27	837412	2·22	860922	1·99	976491	4·22	023509	33
28	837546	2·22	860802	1·99	976744	4·22	023256	32
29	837679	2·22	860682	2·00	976997	4·22	023003	31
30	837812	2·22	860562	2·00	977250	4·22	022750	30
31	9·837945	2·22	9·860442	2·00	9·977503	4·22	10·022497	29
32	838078	2·21	860322	2·00	977756	4·22	022244	28
33	838211	2·21	860202	2·00	978009	4·22	021991	27
34	838344	2·21	860082	2·00	978262	4·22	021738	26
35	838477	2·21	859962	2·00	978515	4·22	021485	25
36	838610	2·21	859842	2·00	978768	4·22	021232	24
37	838742	2·21	859721	2·01	979021	4·22	020979	23
38	838875	2·21	859601	2·01	979274	4·22	020726	22
39	839007	2·21	859480	2·01	979527	4·22	020473	21
40	839140	2·20	859360	2·01	979780	4·22	020220	20
41	9·839272	2·20	9·859239	2·01	9·980033	4·22	10·019967	19
42	839404	2·20	859119	2·01	980286	4·22	019714	18
43	839536	2·20	858998	2·01	980538	4·22	019462	17
44	839668	2·20	858877	2·01	980791	4·21	019209	16
45	839800	2·20	858756	2·02	981044	4·21	018956	15
46	839932	2·20	858635	2·02	981297	4·21	018703	14
47	840064	2·19	858514	2·02	981550	4·21	018450	13
48	840196	2·19	858393	2·02	981803	4·21	018197	12
49	840328	2·19	858272	2·02	982056	4·21	017944	11
50	840459	2·19	858151	2·02	982309	4·21	017691	10
51	9·840591	2·19	9·858029	2·02	9·982562	4·21	10·017438	9
52	840722	2·19	857908	2·02	982814	4·21	017186	8
53	840854	2·19	857786	2·02	983067	4·21	016933	7
54	840985	2·19	857665	2·03	983320	4·21	016680	6
55	841116	2·18	857543	2·03	983573	4·21	016427	5
56	841247	2·18	857422	2·03	983826	4·21	016174	4
57	841378	2·18	857300	2·03	984079	4·21	015921	3
58	841509	2·18	857178	2·03	984331	4·21	015669	2
59	841640	2·18	857056	2·03	984584	4·21	015416	1
60	841771	2·18	856934	2·03	984837	4·21	015163	0
	Cosine	D.	Sine	46°	Cotang.	D.	Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9·841771	2·18	9·856934	2·03	9·984837	4·21	10·015163	60
1	841902	2·18	856812	2·03	985090	4·21	014910	59
2	842033	2·18	856690	2·04	985343	4·21	014657	58
3	842163	2·17	856568	2·04	985596	4·21	014404	57
4	842294	2·17	856446	2·04	985848	4·21	014152	56
5	842424	2·17	856323	2·04	986101	4·21	013899	55
6	842555	2·17	856201	2·04	986354	4·21	013646	54
7	842685	2·17	856078	2·04	986607	4·21	013393	53
8	842815	2·17	855956	2·04	986860	4·21	013140	52
9	842946	2·17	855833	2·04	987112	4·21	012888	51
10	843076	2·17	855711	2·05	987365	4·21	012635	50
11	9·843206	2·16	9·855588	2·05	9·987618	4·21	10·012382	49
12	843336	2·16	855465	2·05	987871	4·21	012129	48
13	843466	2·16	855342	2·05	988123	4·21	011877	47
14	843595	2·16	855219	2·05	988376	4·21	011624	46
15	843725	2·16	855096	2·05	988629	4·21	011371	45
16	843855	2·16	854973	2·05	988882	4·21	011118	44
17	843984	2·16	854850	2·05	989134	4·21	010866	43
18	844114	2·15	854727	2·06	989387	4·21	010613	42
19	844243	2·15	854603	2·06	989640	4·21	010360	41
20	844372	2·15	854480	2·06	989893	4·21	010107	40
21	9·844502	2·15	9·854356	2·06	9·990145	4·21	10·009855	39
22	844631	2·15	854233	2·06	990398	4·21	009602	38
23	844760	2·15	854109	2·06	990651	4·21	009349	37
24	844889	2·15	853986	2·06	990903	4·21	009097	36
25	845018	2·15	853862	2·06	991156	4·21	008844	35
26	845147	2·15	853738	2·06	991409	4·21	008591	34
27	845276	2·14	853614	2·07	991662	4·21	008338	33
28	845405	2·14	853490	2·07	991914	4·21	008086	32
29	845533	2·14	853366	2·07	992167	4·21	007833	31
30	845662	2·14	853242	2·07	992420	4·21	007580	30
31	9·845790	2·14	9·853118	2·07	9·992672	4·21	10·007328	29
32	845919	2·14	852994	2·07	992925	4·21	007075	28
33	846047	2·14	852869	2·07	993178	4·21	006822	27
34	846175	2·14	852745	2·07	993430	4·21	006570	26
35	846304	2·14	852620	2·07	993683	4·21	006317	25
36	846432	2·13	852496	2·08	993936	4·21	006064	24
37	846560	2·13	852371	2·08	994189	4·21	005811	23
38	846688	2·13	852247	2·08	994441	4·21	005559	22
39	846816	2·13	852122	2·08	994694	4·21	005306	21
40	846944	2·13	851997	2·08	994947	4·21	005053	20
41	9·847071	2·13	9·851872	2·08	9·995199	4·21	10·004801	19
42	847199	2·13	851747	2·08	995452	4·21	004548	18
43	847327	2·13	851622	2·08	995705	4·21	004295	17
44	847454	2·12	851497	2·09	995957	4·21	004043	16
45	847582	2·12	851372	2·09	996210	4·21	003790	15
46	847709	2·12	851246	2·09	996463	4·21	003537	14
47	847836	2·12	851121	2·09	996715	4·21	003285	13
48	847964	2·12	850996	2·09	996968	4·21	003032	12
49	848091	2·12	850870	2·09	997221	4·21	002779	11
50	848218	2·12	850745	2·09	997473	4·21	002527	10
51	9·848345	2·12	9·850619	2·09	9·997726	4·21	10·002274	9
52	848472	2·11	850493	2·10	997979	4·21	002021	8
53	848599	2·11	850368	2·10	998231	4·21	001769	7
54	848726	2·11	850242	2·10	998484	4·21	001516	6
55	848852	2·11	850116	2·10	998737	4·21	001263	5
56	848979	2·11	849990	2·10	998989	4·21	001011	4
57	849106	2·11	849864	2·10	999242	4·21	000758	3
58	849232	2·11	849738	2·10	999495	4·21	000505	2
59	849359	2·11	849611	2·10	999748	4·21	000253	1
60	849485	2·11	849485	2·10	10·000000	4·21	10·000000	0
	Cosine	D.	Sine	45°	Cotang.	D.	Tang.	M.

PLANE & SPHERICAL

TRIGONOMETRY

BY EDWARD BROOKS, A.M. PH.D.